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Nuclear Reactions and Level Widths*

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IT is useful, over a wide range of energy of the incident particle, to consider a nuclear reaction as being the succession of two steps. The first step is the formation of the compound nucleus:

Initial nucleus + Incident particle →
Compound nucleus.

The second step is the disintegration of the newly formed compound nucleus into final products:

Compound nucleus →
Final nucleus + Outgoing particle.

A similar division into two steps has often been proposed for chemical reactions and for reactions of electrons with the electronic shell. However, the picture of an intermediate compound state has not proved as useful for these as it has for nuclear reactions. The reason is that only in the latter case have the compound states a sufficiently long lifetime, for a large range of collision energy, to be worthy of being considered as separate entities.

The lifetime τ of a compound state controls, by means of Heisenberg's well-known uncertainty principle, the width $\Gamma=\hbar/\tau$ of that state, where $\hbar=h/2\pi$. The above statement concerning the long lifetime of the compound state is therefore equivalent to the statement that the energy of the compound states is very well defined, that their width Γ on the energy scale is very small compared with their distance D from each other. They are surpassed in this respect only by the ordinary excited states of atoms or nuclei which are also compound states, compound states formed by the absorption of a light quantum rather than the absorption of a particle and which also disintegrate by the emission of a light quantum rather than by the emission of a particle.[1]

Although the above picture of nuclear processes, through the formation and subsequent disintegration of an intermediate compound state, is physically vivid and intuitively reasonable,[2] the mathematical description which is now customary does not follow it very closely. Instead of the above succession of two events, it is customary to consider a stationary state in which we have a stream of incident particles which build up the compound state at the same rate at which it disintegrates into product nuclei. Such a procedure, which was proposed by Kapur and Peierls,[3] is not contradictory to the picture given before; it only describes it from a different point

* Based on an address given on June 21, 1948, at the Madison, Wis., meeting of the American Physical Society.

[1] The conditions for the validity of the picture of the compound nucleus have been carefully analyzed in the doctoral dissertation of L. Eisenbud, Princeton University, 1948.

[2] It has been proposed simultaneously by N. Bohr [*Nature* **137**, 344 (1936)] in his beautiful general discussion of the most important features of nuclear reactions and by G. Breit and E. Wigner (*Physical Rev.* **49**, 519, 1936) in their calculation of the energy dependence of the absorption of slow neutrons.

[3] P. L. Kapur and R. Peierls, *Proc. Roy. Soc.* **A166**, 277 (1938); see also H. A. Bethe, *Rev. Mod. Physics* **9**, 69 (1937).

of view. The latter point of view can be called that of stationary states because one considers a steady flow of incoming particles striking the initial nucleus and an equally large steady flow of reaction products leaving the system.

The main purpose of the present article is to review and correlate some of the information which has accumulated in the past ten years or so on the connection between nuclear reaction rates and level widths. An understanding of this connection and a knowledge of the order of magnitude of the level widths and level densities and of the factors influencing them will be helpful for describing nuclear reactions and for estimating their rates. Of course, the scope of the present article precludes a treatment as thorough and many-sided as Bethe's earlier review[3] was, and we will have to restrict ourselves to a few special aspects of the problem.

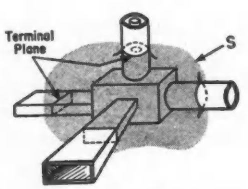

FIG. 1. A transmission line (wave guide) junction.

Valuable experimental material on nuclear reactions has been contributed during the past ten years from almost every part of the world. However, I shall have occasion to refer most often, as far as high energy reactions are concerned, to the results obtained by Scherrer's group in Zurich, by Lauritsen's team at the California Institute of Technology, by Goldhaber at the University of Illinois, by Herb, Barschall, Richards and others at the University of Wisconsin, and by the groups of investigators at the Oslo Laboratory in Norway and at the Argonne and Oak Ridge National Laboratories of the U. S. Atomic Energy Commission. Among the slow neutron experiments, in addition to those performed at the two Atomic Energy Commission's laboratories, the work at Columbia and at Cornell Universities is broadest in scope. Of course, this is only a partial list of references and there is hardly a laboratory interested in nuclear physics that has not contributed to the picture that is beginning to emerge.[4] On the theoretical side also, most of what I write will be a summary of work done by others, notably by Breit[5] at Wisconsin, by Weisskopf[6] and his collaborators at Massachusetts Institute of Technology and by Bethe[7] at Cornell. In addition, as we proceed, it will become clear that many of the concepts and much of the mathematical treatment has appeared in one form or another in radio engineering work.

In particular, G. A. Campbell, O. J. Zobel, R. M. Foster and W. Cauer, together with many others, have anticipated much of the mathematics that I shall review.[8] My most immediate debt in this respect is, however, due my own collaborators and colleagues, Messrs. Eisenbud, Goertzel, Dicke, Teichman and others, to whom I owe not only definite results, as they usually appear in publications,[9] but also hints and corrections of a minor but necessary character.

Impedance and Collision Matrix

I should begin with a short theoretical introduction to explain the connection between nuclear reactions and radio signal transmission. Usually both quantum mechanics and the electromagnetic theory are formulated as causal theories; in a typical case, the wave function or the electromagnetic field is given for a definite time as an initial condition and the same quantities are then calculated for a later time. The method of calculation can then be called the content of the

[4] Some of the references to experimental material will be quoted after Table I.

[5] G. Breit, *Physical Rev.* **58**, 506, 1068 (1940); **69**, 472 (1946). These articles contain references to earlier literature.

[6] V. Weisskopf, *Physical Rev.* **52**, 295 (1937); V. Weisskopf and D. H. Ewing, *Physical Rev.* **57**, 472 (1940); H. Feshbach, D. C. Peaslee and V. F. Weisskopf, *Physical Rev.* **71**, 145 (1947).

[7] H. A. Bethe, *Rev. Mod. Physics* **9**, 71 (1937), especially pages 101–117. E. J. Konopinski and H. A. Bethe, *Physical Rev.* **54**, 130 (1938); H. A. Bethe, *Physical Rev.* **57**, 1125 (1940).

[8] R. M. Foster, *Bell System Tech. Journ.* **3**, 259 (1924); G. A. Campbell, *Bell System Tech. Journ.* **1**, No. 2, p. 1 (1922); O. J. Zobel, *ibid.* **2**, 1 (1923); W. Cauer, *Preuss. Akad. Ber.* 1931, p. 30; *Physics* **2**, 242 (1932). *Cf.* also H. W. Bode, *Network analysis and feedback amplifier design* (Van Nostrand, 1945); also, forthcoming article of R. H. Dicke in *Massachusetts Institute of Technology Radiation Laboratory Series* (McGraw-Hill, 1947–48), and unpublished notes of J. Schwinger, "Theory of Obstacles in Resonant Cavities and Wave Guides," 1943.

[9] L. Eisenbud and E. P. Wigner, *Physical Rev.* **72**, 29 (1947); G. Goertzel, *Physical Rev.* **73**, 1463 (1948); E. P. Wigner, *Physical Rev.* **73**, 1002 (1948).

theory. It gives the system's state at a later time from the knowledge of the system's state at an earlier instant. Such a formulation of the laws of physics is in many ways the most natural one; it certainly is the traditional one.

However, other formulations are possible also. Instead of giving the state of the system throughout space at one instant of time as the initial condition, one can give part of the state of the system, for instance, a description of all incoming waves, for all times. The question then concerns the rest of the system's state at all times; in the example above, the problem would be to describe the outgoing waves. Formulations of the laws of nature which refer to questions of this second kind are perhaps less customary but, as Heisenberg[10] in particular has emphasized, not necessarily less fundamental. Furthermore, even though the first kind of question is the more usual one, large parts of physics are more concerned with the second type of question. Thus, for example, in optics, the typical experimental arrangement contains a light source giving a constant beam of light incident upon the optical apparatus. The question concerns the intensity of the light at such parts of space as are illuminated by light passing through this apparatus; that is, it concerns essentially the waves which go out of the apparatus.

Problems connected with radio circuits are of similar nature. The optical apparatus is replaced in this case by what has been called, very pictorially, a black box, but which now usually goes under the more dignified name of wave guide junction. Figure 1 shows such a wave guide junction. It is a drawing by R. H. Dicke from whom I also learned most of what I shall say about it. The wave guides joined in the junction partly bring in signals, partly carry them out. The problem is to calculate the outgoing signals if the inflowing signals are known.

Radio engineers represent junctions between interdependent circuits by diagrams like that in Fig. 2. Points on the transmission lines are selected and called terminals of that line. There are four such terminals in Fig. 2. The voltage at any particular terminal is bound to depend on the currents at all the terminals. The relation between the voltage V_s at a particular terminal s, and the currents $i_{s'}$, at all terminals including s, is given by

$$V_s=\sum_{s'} Z_{ss'}i_{s'},$$

where $Z_{ss'}$ are the elements of the impedance matrix. The summation extends over all terminals, that is, over all values of s'.

The picture of the wave guide junction also permits one to visualize the events in configuration space which are important in a collision. The "black box" itself corresponds to that part of configuration space about which we know very little: it is Eisenbud's *internal region* in which the colliding particles are close together, interacting

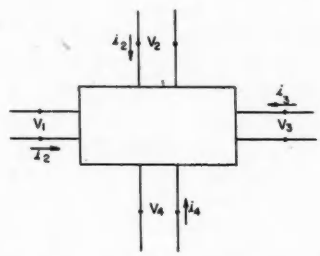

FIG. 2. A junction of interdependent electric circuits.

strongly. The different wave guides correspond to alternate ways in which the collision system can be formed as well as the various ways in which it can disintegrate. For instance, a wave coming in through the rectangular guide at the left (Fig. 1) may correspond to a proton wave hitting a Li^7 nucleus, and an outgoing wave in the *same* guide to the disintegration of the compound nucleus into a Li^7 and a proton. The vertical round guide may correspond to the pair Li^6+H^2 which can form the same compound nucleus, again the incoming wave corresponding to formation, and the outgoing wave to the disintegration of the complex. Of course, this three-dimensional representation of the collision, which actually takes place in many-dimensional configuration space, is merely schematic. It can serve, however, to give a lively picture of several important aspects of nuclear collision phenomena.

[10] W. Heisenberg, *Zeits. f. Physik* **120**, 513, 673 (1943); *Zeits. f. Naturf.* **1**, 608 (1946); also H. A. Kramers, *Hand und Jahrbuch Chem. Physik*, Vol. I.

Generalization of the Ladenburg-Kramers-Heisenberg Dispersion Formula

Before continuing, it should be reiterated that most of what has been said so far and most of what is to be said in the remainder of this article is not new, except perhaps in its generality, and that similar ideas have been developed independently in many fields—notably in acoustics and in electroacoustics. Two characteristics of all these developments are: (a) a division of space into two regions—the inside and outside of the black box—by an arbitrary surface, and (b) a belief that what is outside, that is, what is in the external region is well understood and can be mastered mathematically. On the other hand, the inside of the black box, the internal region, can be described only globally, by giving the conditions which it imposes on the wave function or the electromagnetic field at the surface S which divides the two regions. In other words, the result of the application of the laws of nature within the internal region will manifest itself only by imposing boundary conditions on the field quantities at its surface. An advantage of this type of formulation of the laws of physics is that it permits hiding our ignorance about the internal region. This ignorance may be caused both by an actual lack of knowledge of the laws of physics as they apply in nuclear collisions and by inadequate mathematical mastery of the complicated situation in the internal region.

Let us now look at the various ways of expressing the boundary conditions which are imposed by the internal region on the behavior of the wave function on the surface S. One can say that these boundary conditions lead to a description of the outgoing waves in terms of the incoming waves. The connecting link between these two types of waves is the collision matrix.[11] Unfortunately, this matrix, which contains the physically important quantities in the most direct fashion, depends on the energy in a rather complicated way. Another way in which the boundary condition can be expressed is to consider the value of the wave function on S to be determined by the normal derivative of the wave function on the same surface. The second procedure corresponds to the admittance and impedance matrix method of radio engineering. We shall call the matrix in question the derivative matrix R. It expresses the amplitudes v_s of the waves corresponding to the various reaction products s in terms of the normal derivatives d_s of these waves at S in accordance with the relation:

$$v_s = \sum_{s'} R_{ss'} d_{s'}. \tag{1}$$

In Eq. (1) s and s' refer to the different channels in configuration space through which incident particles can enter and outgoing particles can leave the compound nucleus. These channels are quite analogous to the wave guides of Figs. 1 and 2 and Eq. (1) is the analog of the equation connecting the voltages V_s and currents $i_{s'}$ at all terminals.

Equation (1) expresses the boundary conditions on S but has less immediate physical significance than the collision matrix. Even if R is known, the problem is not yet entirely solved because the real problem involves incoming and outgoing waves through S rather than values and derivatives of the wave function at that surface. However, if we really know the properties of the solutions of the wave equation in the external region, Eq. (1) contains all the needed information because the solution can always be continued into the external region if its value and its derivative on S are known.

The great advantage of the R matrix is that its energy dependence follows the very simple rule:

$$R_{ss'}(E) = \sum_{\lambda} \frac{\gamma_{\lambda s}\gamma_{\lambda s'}}{E_\lambda - E}. \tag{2}$$

In the right-hand member of this equation, all the energy dependence is given by E in the denominator, both the resonance energies E_λ and the transition strengths $\gamma_{\lambda s}$ being independent of energy. Multiplied by twice the wave number, the square of $\gamma_{\lambda s}$ gives the neutron width of the level if s corresponds to the emission of a neutron. If s corresponds to the emission of a proton, $\gamma_{\lambda s}^2$ is what Christy[12] calls the proton width without barrier. It has a similar meaning in more complicated cases. Equation (2) not only has some

[11] J. A. Wheeler, *Physical Rev.* **52**, 1107 (1937); P. L. Kapur and R. Peierls, *Proc. Roy. Soc.* **A166**, 277 (1938).

[12] R. F. Christy and R. Latter, *Rev. Mod. Physics* **20**, 158 (1948).

similarity to the Ladenburg-Kramers-Heisenberg dispersion formula but can actually be considered to be a generalization thereof.[13] Just as the LKH formula describes the dispersion in terms of the energies of the stationary states and f values, so Eq. (2) describes the nuclear reaction in terms of E_λ and $\gamma_{\lambda s}$.

There are, unfortunately, two rather far reaching differences between the quantities occurring in the Ladenburg-Kramers-Heisenberg dispersion formula and the quantities E_λ, $\gamma_{\lambda s}$ of Eq. (2). The first of these differences is that the E_λ and $\gamma_{\lambda s}$ are only approximately independent of the position of the arbitrary surface S, while the quantities in the dispersion formula, due to the special conditions of that phenomenon, are, for all practical purposes, independent of any such artificiality. The second difference is a consequence of the first: E_λ and $\gamma_{\lambda s}$, since they depend on as artificial a circumstance as the position of S, cannot have as deep and as direct a physical significance as the constants of the dispersion formula. It remains an open question, therefore, whether, and to what extent, it will be useful to characterize the matrix function R and the collision matrix by E_λ and $\gamma_{\lambda s}$. It is reassuring, perhaps, in this connection, that the variation of these quantities with the position of the surface S is given by simple differential equations. The rest of our discussion will be in terms of these quantities, in the hope that the laws governing them are simpler than those giving the collision matrix directly.

Level Spacing

Very little that is new can be said about the spacing of levels in addition to the empirical and theoretical rules that have already been obtained by Weisskopf, Bethe and others. The summary of Hornyak and Lauritsen[14] perhaps indicates that, as far as very light nuclei are concerned, the level spacing may have been underestimated and may better be represented by the equation

$$D=2\exp(-\tfrac{1}{2}E^{\frac{1}{2}}) \tag{3}$$

where all energies are expressed in Mev. However, since it is always possible and, as we shall see, perhaps even probable, that more levels have been missed than is apparent, the above conclusion may be faulty. I would like here to remark that it would seem probable that a careful survey of the levels in light elements would give a more accurate check on the charge-independent or charge-dependent nature of nuclear forces than could be obtained from direct measurements. I understand, however, that such measurements are already under consideration at the University of Wisconsin.

Although the spacing of levels in light nuclei is of the order of hundreds of kilovolts, yet in the heaviest nuclei it does not amount to more than a few volts. The results obtained by using the various velocity selectors and also by other techniques indicate that the level spacing in the slow neutron region is practically constant between atomic numbers 100 and 200. This is quite surprising at first sight. It may well be, however, that the effect of the increase of the total number of particles is compensated by the decrease of the binding energy of the neutron. On this account, the compound formed by a cadmium nucleus and a neutron has more excitation energy than the compound formed, say, by an iridium nucleus and a neutron. The effect of the increase of the neutron's binding energy on the level density demonstrates itself directly and rather dramatically; practically all strong slow neutron absorbers have odd mass numbers. Since the average slow neutron absorption is inversely proportional to the square of the level spacing and since the binding energy of a neutron to odd nuclei is about 1 Mev higher than to even nuclei, the result is actually what might have been expected.

Level Widths

We now turn to the quantities $\gamma_{\lambda s}$ about which there is more recent information than about level spacing. The square of $\gamma_{\lambda s}$ is the reduced transition probability of the compound state E_λ into two separated nuclei denoted by s. The phrase, reduced transition probability, refers to that factor in the transition probability which depends only on the internal region; in order to obtain the actual transition probability or, what is the same thing, the partial width, $\gamma_{\lambda s}^2$ still has to be

[13] For the Ladenburg-Kramers-Heisenberg dispersion formula see A. Rubinowicz' article in Geiger-Scheel's *Handbuch der Physik* (Berlin, 1933), Vol. 24, p. 63f.

[14] W. F. Hornyak and T. Lauritsen, *Rev. Mod. Physics* **20**, 191 (1948).

TABLE I. Nuclear energy level widths and transition probabilities.

Reaction	Level	Spacing D	Γ	$\gamma^2\times10^{13}$	$\gamma^2\times10^{13}$ / spacing	Γ_r (ev)	Ref.
Li^7+H	440 kev	1000 kev	12 kev	140 kev cm	0.15 cm	20	a
				1200 "	1.2	20	t
Be^9+H	305 "	500 "	140 "	6000 "	12	—	b
Be^9+H	988 "	500 "	95 "	1800 "	3.5	30	c
Be^9+H	1077 "	500 "	4 "	76 "	0.15	6	c
$C^{12}+H$	450 "	500 "	35 "	3400 "	8	0.6	d
$C^{13}+H$	550 "	500 "	40 "	3000 "	6	20	d
$O^{16}+n$	450 "	500 "	200 "	700 "	1.4	—	s
$Al^{27}+n$	155 "	100 "	20 "	125 "	1.25	1	e
$Mn^{55}+n$	0.3 "	0.5 "	$\sim$5 ev	0.7 "	1.4	$\sim$2	f, k
$Ni^{58}+n$	15 "	50 "	4 kev	75 "	1.5		g, m
$Co^{59}+n$	0.11 kev	1 "	4 ev	0.9 "	0.9	0.2	h, l
$Cd^{113}+n$	0.18 ev	50 ev	0.8×10^{-3} ev	4.5 ev cm	0.09	0.1	f, i, p
$In^{115}+n$	1.44 "	10 "	2 "	4 "	0.4	(0.1)	q, j
$I^{127}+n$	20 "	20 "	0.8 "	0.4 "	0.02	(0.1)	n, h
$Eu^{153}+n$	0.54 "	10 "	2.5 "	7.5 "	0.75	(0.1)	r, o
$Ta^{181}+n$	4 "	10 "	1.4 "	1.6 "	0.8	(0.1)	j
	10 "	10 "	1.9 "	1.4 "	0.7	(0.1)	j
	13 "	10 "	0.3 "	0.2 "	0.01	(0.1)	j
$Ir+n$	0.64 "	6 "	0.3 "	0.9 "	0.15	0.07	f, i
	1.27 "	6 "	0.55 "	1.1 "	0.2	0.07	f, i
Tl	270 "	1000 "	$4.6/\Gamma_r$ "	$320/\Gamma_r$ "	$0.32/\Gamma_r$		h

[a] L. R. Hafstad, N. P. Heydenberg and M. A. Tuve, *Physical Rev.* **50**, 504 (1936); C. M. Hudson, R. G. Herb and G. J. Plain, *Physical Rev.* **57**, 587 (1940); J. E. Evans and T. W. Bonner, *Bull. Am. Physical Soc.* **22**, No. 5, p. 7 (1947), *Physical Rev.* **72**, 163*A* (1947).

[b] N. Hole, J. Holtsmark and R. Tangen, *Naturw.* **28**, 335 (1940) (Professor R. F. Christy's interpretation).

[c] W. J. Hushley, *Physical Rev.* **67**, 34 (1945); T. Lauritsen and W. A. Fowler, *Physical Rev.* **72**, 739 (1947). Since the p, α and p, d reactions do not show resonances at 988 and at 1077 kev, it is inferred that the compound state is a $J=0$ state of even parity. Such a state cannot disintegrate into either Li^6+He^4 or Be^8+H^2. It can be formed from Be^9+H^1 only if the proton has angular momentum 1 if Be^9 has an odd normal state. (If the normal state of Be^9 were even, the angular momentum of the proton would have to be 2, further increasing the γ^2.)

[d] J. O'Reilly and R. F. Christy, *Bull. Am. Physical Soc.* **23**, No. 1, 10 (1948); W. A. Fowler, C. C. Lauritsen, and T. Lauritsen, Rev. Mod. Physics **20**, 236 (1948).

[e] L. W. Seagondollar and H. H. Barschall, *Physical Rev.* **72**, 439 (1947).

[f] L. J. Rainwater, W. W. Havens, C. S. Wu and J. R. Dunning, *Physical Rev.* **71**, 65 (1947).

[g] H. H. Barschall, C. K. Bockelman and L. W. Seagondollar, *Physical Rev.* **73**, 659 (1948).

[h] C. S. Wu, L. J. Rainwater and W. W. Havens, *Physical Rev.* **71**, 174 (1947). The Co resonance has been reinterpreted as a scattering resonance. Cf. reference 1.

[i] R. B. Sawyer, E. O. Wollan, S. Bernstein and K. C. Peterson, *Physical Rev.* **72**, 109 (1947).

[j] W. W. Havens, C. S. Wu, L. J. Rainwater and C. L. Meaker, *Physical Rev.* **71**, 165 (1947).

[k] F. G. P. Seidl, S. P. Harris and A. S. Langsdorf, *Physical Rev.* **72**, 168 (1947); M. Goldhaber and A. A. Yalow, *Physical Rev.* **69**, 47 (1946).

[l] S. P. Harris, A. S. Langsdorf and T. G. P. Seidl, *Physical Rev.* **72**, 866 (1947).

[m] W. W. Havens, L. J. Rainwater, C. S. Wu and J. R. Dunning, *Physical Rev.* **73**, 963 (1948).

[n] Wm. B. Jones, *Physical Rev.* **72**, 362 (1947).

[o] W. J. Sturm, *Physical Rev.* **71**, 757 (1947).

[p] C. P. Baker and R. F. Bacher, *Physical Rev.* **59**, 332 (1941); W. H. Zinn, *Physical Rev.* **71**, 752 (1947).

[q] B. D. MacDaniel, *Physical Rev.* **70**, 832 (1946).

[r] L. B. Borst, A. J. Ulrich, C. L. Osborne and B. Hasbrouck, *Physical Rev.* **70**, 557 (1946).

[s] D. H. Frisch, unpublished, Bretscher and Murrell, unpublished but quoted by H. H. Goldsmith, H. W. Ibser and B. T. Feld, *Rev. Mod. Physics* **19**, 259 (1947). Professor H. H. Barschall kindly informs me that, according to more recent measurements, the width of the 450-kev level is only 45 kev. However, he also concludes that the neutrons have angular momentum 1 instead of 0 as was assumed in the table. The two changes compensate so that the value for γ^2 given in the table remains valid.

[t] Professor R. F. Christy kindly informs me that recent evidence obtained at the California Institute of Technology indicates that the proton has an angular momentum 1 at this resonance. This would increase the reduced width γ^2 as indicated.

multiplied by a factor which depends on the behavior of the wave function in the external region. In the case of neutron emission, this factor is simply twice the wave number. This leads to the familiar result that the neutron width is, *ceteris paribus*, proportional to the velocity of the escaping neutron. If charged particles result from the reaction, the second factor to the level width contains Gamow's barrier penetration factor. We shall not concern ourselves with the effect of this second factor on the particle width since it is well understood, but will center our attention on the first factor, $\gamma_{\lambda s}{}^2$.

This $\gamma_{\lambda s}{}^2$ will depend on several quantities: first, on the state of excitation λ of the compound nucleus; second, on the type of particle that is being emitted, whether it is a neutron, a proton, or an α-particle; third, on the type and state of excitation of the nucleus left behind; and fourth, on the kinetic energy with which the particles separate.

The dependence on λ, that is, on the state of excitation of the compound nucleus, has been investigated in detail. Years ago, in 1935, Bethe showed that the quantities $\gamma_{\lambda s}{}^2$ should be inversely proportional to the level density. His considerations, which apply best to the high energy region, have been extended recently by Feshbach, Peaslee

and Weisskopf[15] whose work has given the theory added interest and meaning. Table I gives a survey of the experimental material collected principally by T. Teichmann.

The first column contains s, representing the pair of particles whose formation gives rise to the width in question. One sees that the first few examples refer to proton emission, the rest to neutron emission. The second column gives the position of the level, or, more accurately, the kinetic energy which these particles acquire as a result of the disintegration of the compound nucleus. This kinetic energy ranges from a few hundred kilovolts down to a volt or so. The third column contains an estimate of the spacing of the levels, or the distance of the various E_λ from each other. These numbers have no really exact meaning and had to be largely estimated. They range from a thousand kilovolts down to a few volts. The fourth column contains the actual level width with respect to the emission of a particle which is more or less a directly observed quantity.

The guiding principle in the selection of the experimental material for Table I was a need for the knowledge of the angular momentum of the colliding particles. This information is necessary in order to calculate the reduced transition probability $\gamma_{\lambda s}{}^2$ from the observed width of the level. The information in question is most easily available at low energies where only low angular momenta play a role. The majority of the resonance levels of Table I have a relatively low energy. In spite of this, it is possible that the estimates may be inaccurate in some cases and that, as a result, the reduced widths are larger than we assumed. Such errors are particularly likely if the energy given in the second column is more than, say, 100 kev. Toward the end of Table I, where the excitation energies are of the order of dozens of volts rather than many kilovolts, the figures are quite reliable.

The reduced width, obtained from the total width by eliminating the penetration factor and other factors originating in the external region, is given in the fifth column. It will be noted that the different widths quoted vary by a factor of more than a million. The sixth column gives the ratio of the reduced particle width to the level spacing. This ratio should be constant according to the Bethe-Feshbach-Peaslee-Weisskopf theory. We see, however, that this is true only in the sense that the ratios given are all of the same order of magnitude. The last numerical column gives the width with respect to the emission of radiation of the level in question.

Considering the approximate nature of many of the data the most remarkable feature of Table I is the very small amount of variation which the sixth column exhibits. This is the great success of the Bethe-Weisskopf theory: the quantities γ^2 and the spacings both vary by factors of several hundred thousand; their ratio, however, is nearly constant. The second remarkable feature of the table is that γ^2/D is not quite constant, but exhibits irregular fluctuations as well as a general trend toward lower values at the end of the table where heavy elements are involved. The first point is not surprising, even though it is somewhat obscured by Weisskopf's rather schematic theory. Every state λ is, so to say, an individuality, whose wave function has its own peculiarities and character. It would be surprising to find a great uniformity in the probability of disintegration from such widely different initial states into a definite final state. The situation is different with respect to radiation widths, at least in the lower part of the table. Here, the individuality of the levels λ is compensated by the large variety of final states in which the residual nucleus can remain after the emission of the first γ quantum. As a result, the radiation widths are all quite uniform beyond the mass number about a hundred.

It may be worth while to remark here that the derivative matrix R gives a very simple interpretation to the sixth column of Table I. It is clear that every matrix element R_{st} of R is an analytic function of E; that is, it can be extended into the complex plane. This is closely related to the analytic nature of the collision matrix which was the subject of numerous investigations by Heisenberg, Moeller and Kramers.[16]

[15] H. Feshbach, D. C. Peaslee and V. F. Weisskopf, *loc. cit.*, reference 6.

[16] W. Heisenberg, *loc. cit.*, reference 10. C. Moeller, *Kgl. Danske Vid. Sels. Mat. Phys. Med.* **23**, 1 (1945); *Nature* **158**, 403 (1946). Cf. also Ning Hu, *Physical Rev.* **74**, 131 (1948). It seems to the present writer, however, that Hu's claim for a more rigorous treatment than contained in the articles of reference 9 would be hard to justify. As to the analytic nature of the collision matrix, this is an obvious consequence of the analytic nature of R, if R is one-dimensional,

Let us consider, in particular, the diagonal elements R_{ss}. These undergo very rapid variations for real values of E. They go from $-\infty$ to $+\infty$ between any two neighbors of resonance levels. For complex values of E, they are much smoother functions. In particular, if we choose the imaginary part of E large compared to the level spacing, the imaginary part of R_{ss} becomes π times the average reduced width divided by the level spacing—the average being taken over a region about as wide as the imaginary part of E. The real part of R_{ss} drops more rapidly for complex values of E. It follows that the quantities R_{ss}, which are intensely fluctuating real functions for real values of E, become rather smooth and imaginary functions for complex values of E. In this case the absolute value of R_{ss} is just π times the value of the sixth column of Table I. The theories which were mentioned before can therefore be formulated alternately by the statement that if s corresponds to the emission of a particle from a not too highly excited nucleus, the quantities R_{ss} should have for all nuclei the same values for imaginary values of E. The non-diagonal elements R_{st} are probably all much smaller than the R_{ss}.

Even though the fluctuations of the $\gamma_{\lambda s}$ are not surprising, they pose a very definite problem—the determination of their magnitude. This is an interesting quantity from a theoretical point of view, and an important one from a practical point of view. If I read the Smyth report correctly, the possibility of a slow neutron chain reaction is materially enhanced, if not conditioned, by such a fluctuation.

These fluctuations may also explain, at least partially, a discrepancy which one finds if one tries to calculate the average fast neutron absorption from the data of Table I. One is led, then, apparently, to very large radiation widths. In some cases, such as the $C^{13}+H$ reaction, there are only a few excited states within a reasonable distance from the normal state. In such cases a single large transition probability, though surprising from the point of view of our present concepts of nuclear structure, is not in direct contradiction to any well established general principle, such as the f-sum rule. One is more reluctant, however, to assume similar radiation widths in heavier elements where the statistical picture should be certainly applicable.

Before going over to the discussion of the experimental material on the absorption of fast neutrons, a few remarks concerning the dependence of the $\gamma_{\lambda s}$ on the second variable, or the state of the excitation s of the residual nucleus, may be well in place. The first and perhaps most natural assumption[6] is that the $\gamma_{\lambda s}$ are independent of the state s, that the state of excitation of the residual nucleus is determined solely by statistical factors, such as the volume available in phase space. Indeed, many experiments can be interpreted quite satisfactorily on this basis. A notable exception is the ratio of the yield of photoneutrons to that of photoprotons for which the experiments of Hirzel and Wäffler[17] gave a large but in many cases a more than a hundred times smaller value than the statistical theory. According to the latter, a very large number of low energy neutrons should be ejected by the γ-rays leaving the residual nucleus highly excited. The escape of the corresponding low energy protons is prevented by the Coulomb barrier. As a result of this, the theoretical ratio of photoneutrons to photoprotons appears to be very large and considerably in excess of the experimental ratio. The explanation of this discrepancy, as put forward by Schiff,[18] assumes that the number of cases in which the residual nucleus is left behind in a highly excited state is, both in the γ, n and in the γ, p reaction, very much diminished by a decrease of the transition probability into highly excited states. This means that the $\gamma_{\lambda s}$ are quite small for s which correspond to such states.

This is indeed what one has to expect. Just as the $\gamma_{\lambda s}$, for given s, decrease with increasing λ because of the increasing complexity of the resonance states λ, in a similar way will they decrease, for given λ, with increasing s, because of the increasing complexity of the wave function of the residual nucleus. A compact mathematical expression for both facts is given by sum rules which are analogous to the well-known sum rules

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that is, if only scattering is possible. If a reaction is possible as well, both the derivative matrix R and the collision matrix have several dimensions and the matrix elements of the latter are clearly nonanalytic while those of R remain analytic. (Cf. Eq. (2).)

[17] O. Hirzel and H. Wäffler, *Helv. Physica Acta* **20**, 374 (1947).

[18] L. I. Schiff, *Physical Rev.* **73**, 1911 (1948).

of spectroscopy. However, while the sum rule leading to the proportionality of level density and neutron width is rather difficult to formulate, the sum rule with respect to the s is very simple,

$$\sum_s \gamma_{\lambda s}{}^2 \approx 3\hbar^2/2Ma, \tag{4}$$

where M is the mass of the ejected particle and a is the nuclear radius. This equation shows that the $\gamma_{\lambda s}$ cannot be entirely independent of s: the sum would be infinite if this were the case. A comparison of some of the large γ^2 given in Table I shows that they constitute several percent of the total sum. The question which the above argument leaves open to some degree, but which Schiff attempts to answer in the positive sense, is whether the decrease of the $\gamma_{\lambda s}$ takes place in the relevant energy region to explain the results of Hirzel and Wäffler.

Among the factors which influence the level width, we have not yet discussed the character of the particle which is emitted. There is little experimental material on this question that can be interpreted unambiguously. Let me state, therefore, only in general that there is little reason to believe that the γ^2/D would be different for protons from what it is for neutrons. However, the value of this quantity is probably smaller for deuterons, α-particles and other composite nuclei.

Average Absorption of Fast Neutrons

In many experiments, and in most practical applications, the average of the neutron absorption cross section is as important as its detailed behavior. The fluctuations of the γ, which were discussed in the preceding section, have an important bearing on this.

For the average neutron absorption cross section Breit has already given implicitly the formula[19]

$$\sigma_{Av} = \frac{4\pi^2}{k}\frac{\gamma^2}{D}\frac{\Gamma_r(1+ak)^2}{\Gamma_r+2\gamma^2 k}. \tag{5}$$

Here, k is the wave number of the neutron, γ^2 and Γ_r are the reduced neutron width and the radiation width, respectively, a is the radius of the nucleus, and D is the level spacing, that is, the average distance from each other of levels with a definite total angular momentum and parity. In the derivation of Eq. (5) it was assumed that the levels are widely separated so that every level contributes as much to the integral of the absorption coefficient over the whole range of energy, as if it were alone. The validity of this assumption has been investigated recently by Teichmann who found it well corroborated. He discovered, furthermore, that the correction to Eq. (5), which is small, is negative, so that the actual cross section is smaller than the equation would indicate. In addition, the phenomenon of inelastic scattering is disregarded in Eq. (5). This will introduce another negative correction, though not a very large one in the range of energy about 1 Mev at which most measurements were made. If one expresses k in terms of the energy E in electron volts, uses the same unit for D and barns for σ_{Av}, Eq. (5) becomes

$$\sigma_{Av} = \frac{1800f}{E^{\frac{1}{2}}}\frac{\Gamma_r}{\Gamma_r+4.4\times10^{-4}fDE^{\frac{1}{2}}}\left(1+\frac{A^{\frac{1}{3}}E^{\frac{1}{2}}}{3100}\right)^2, \tag{5a}$$

where A is the mass number and f the value of the sixth column of Table I; it is of the order of magnitude unity.

If the radiation width is larger than the neutron width, as it is at low energies, the average cross section varies as $E^{-\frac{1}{2}}$, as already pointed out by Bethe and experimentally verified in the Argonne Laboratory by Dancoff and Lichtenberger. At higher energies, the cross section is inversely proportional to the first power of E and at even higher energies, when the last factor of the right-hand member of Eq. (5a) becomes important, it is independent of E. If one further increases the energy to values at which inelastic scattering and other phenomena become important, the cross section will drop again.

The transition from the $E^{-\frac{1}{2}}$ to E^{-1} region occurs around 30 kev in heavy elements, and at lower energies in lighter elements. There is no $E^{-\frac{1}{2}}$ region at all in the lightest elements. The E^{-1} region should extend upward to about 300 kev for heavy and to about 1 Mev for light elements. In the E^{-1} region the absolute value of the cross section is $4.1\Gamma_r/DE$ if E is measured in Mev.

It will be noticed that the cross section in the

[19] Cf. reference 2. Also F. Kalckar, J. R. Oppenheimer and R. Serber, *Physical Rev.* 52, 273 (1937). The factor $(1+ak)^2$ is to account for the effect of higher angular momenta.

TABLE II. Neutron widths calculated from experimental cross sections.

Element	σ_{Av}(millib)	$4.1(1+A^{1/3}/3.1)^2$	$10^3\Gamma_r/D$	D	Γ_r/ev
Al^{27}	0.4	16	0.025	$\frac{1}{2}\times$ 100 kev	1.25
Mn^{55}	3.5	20	0.17	$\frac{1}{2}\times$ 0.5 kev	0.04
Ni^{58}	~6	21	~0.28	50 kev	14
Co^{59}	9	21	0.43	$\frac{1}{2}\times$ 1 kev	0.2
Rh^{103}	120	26	4.6	~ 30 ev	0.15
$Ag^{107+109}$	130	26	5	$\frac{1}{2}\times$ 60 ev	0.15
In^{115}	240	27	9	$\frac{1}{2}\times$ 10 ev	0.04
I^{127}	105	28	3.8	$\frac{1}{2}\times$ 20 ev	0.04
$Ir^{191,193}$	200	33	6.1	$\frac{1}{2}\times$ 6 ev	0.02
Au^{197}	120	34	3.5	$\frac{1}{2}\times$ 30 ev	0.05

E^{-1} and in the following region is independent of the neutron width γ^2 and should give, therefore, a direct measure of Γ_r/D. The corresponding calculation is carried out in Table II for $E=1$ Mev at which energy there are available at least approximate experimental values for the cross section.[20] Since the level density increases with increasing energy of the neutron, those values of D of Table I which were obtained from the very low energy region were halved. They are so indicated in Table II.

One sees that most of the widths, Γ_r, obtained in Table II from the 1-Mev absorption, agree reasonably well with the values given in Table I which were obtained directly from the widths of the resonance curves. If anything, the Γ_r of Table II are smaller than those of Table I. As was mentioned before, inelastic scattering would have this effect. However, Al and Ni form exceptions under this rule; Table II gives, particularly in the case of Ni, values that are too large. It should be noted that the level density has been obtained in these two cases from the behavior of neutron scattering at relatively high energies. It seems not unreasonable to assume, therefore, that in addition to the levels which appear in the neutron scattering measurement of Barschall, Bockelman and Seagondollar,[21] further narrower levels are present. Their number would have to be about ten times as great as the number of broad levels so that their spacing would be about 5 kev. Their width could be quite small: at 1 Mev the neutron absorption is practically independent of the neutron width.

One could object to the above assumption on two grounds. First, it is possible that the radiation width may perhaps be actually as large as the 14 ev calculated in Table II. This is, however, most unlikely considering the high excitation of the Ni nucleus in question. Second, one could assume that the maxima in the scattering cross section which appear in the diagram of Barschall are not due to single broad levels but to a multitude of narrow ones. This second assumption is rendered unlikely by the observation that the observed cross section at 15 kev is only about six times smaller than the theoretical maximum. If it is due to several unresolved levels, their spacing would be only about ten times their width and their density would show unreasonably large fluctuations.

One is led, therefore, to the following picture. The γ^2 of the levels shows considerable fluctuations. The absorption at very low energies is caused by the lowest of the levels which has, most probably, not a particularly large γ^2. The scattering maxima at intermediate energies are mostly due to a few levels with particularly large γ^2. On the other hand, practically all the levels contribute equally to the neutron absorption at 1 Mev. Hence, the majority of the levels, most of them with rather small γ^2, account for most of the absorption.

So much about the fluctuations of the γ^2 and the influence of these fluctuations on the average behavior of nuclei. The general trend of the γ^2/D, showing, as they do, a decrease with increasing complexity of the nucleus, is almost as striking as the effects of the fluctuations of the γ^2. Furthermore, the Γ_r exhibits a similar trend. No really satisfactory explanation for these phenomena has been found so far. Among the very lightest elements, one should perhaps not be surprised to find exceptionally large particle and radiative widths. One deals in these cases with some of the lowest levels, to which statistical arguments cannot yet be applied with safety. Toward the middle of Table I we have seen that the observations favor the detection of the wide levels. While these remarks may give a partial explanation of

[20] D. J. Hughes, private communication; also MDDC-27. Cf. also J. H. E. Griffith, *Proc. Roy. Soc.* **170**, 513 (1939); H. V. Halban and L. Kowarski, *Nature* **142**, 392 (1938); S. Kikuchi and H. Aoki, *Physical Math. Soc. Japan* **21**, 75, 232 (1939); C. R. Mescheryakow, *C. R. USSR* **48**, 555 (1945).

[21] H. H. Barschall, C. K. Bockelman and L. W. Seagondollar, *Physical Rev.* **73**, 659 (1948). I am much indebted to Dr. H. Brooks for a number of very helpful remarks on this question.

the trend towards smaller values near the ends of the last two columns of Table I, it is doubtful that they give the whole picture.

A few words may be added about experiments and problems which might be considered particularly important in this field. The desirability of a more complete and more accurate determination of the excited states of light nuclei has already been mentioned. Such measurements would shed light on the equality of proton-proton, proton-neutron and neutron-neutron interactions. It is probably unnecessary to mention that a more extensive survey of level densities, together with a determination of the spin and parity quantum numbers would be most desirable. Such a survey should cover not only the average level densities but also answer the question, raised in particular by M. Goldhaber, as to whether or not the fluctuations of the level density around the average can be explained on a purely statistical basis. The same two questions may well be asked also concerning level widths. Corroboration or a refutation of the apparent deviation from proportionality between level width and level density would be very important and an estimate of the probable deviation of the level density from the average would also be highly desirable. These last questions have not only great theoretical but also some practical significance. The same holds true of an exploration of the phenomena attending M. G. Mayer's "mystic" numbers,[22] in particular an elucidation of the direct cause of the decreased absorption of the "mystic" nuclei. It seems likely that an abnormally low level density is at least partially responsible for the low absorption of these nuclei. If this should be corroborated, the range of this abnormality in energy and mass number will remain to be explored. No reasonable value of Γ_r will permit the data of Wu, Rainwater and Havens[23] on Tl to give an even approximately normal value of γ^2/D.

The writer is very much indebted to Dr. C. Kikuchi for several suggestions which have greatly improved the clarity and readability of this article.

[22] Maria G. Mayer, *Physical Rev.* **74**, 235 (1948).

[23] C. S. Wu, L. J. Rainwater and W. W. Havens, *Physical Rev.* **71**, 174 (1947).

Big Four, by A.P.H.

Mr. Watt, Monsieur Ampere, Signor Volta, and Herr Ohm—
A highly international quartet
Have shed a lot of light on every industry and home,
But, I own, I never understood them yet.

How often have I struggled, just a literary dolt,
To appreciate the meaning of a watt!
How often have they told me what it is about a volt
That an ampere or an ohm hasn't got!

All in vain. I know "electric," as they tell me in a tome,
Is a word that stood for "amber" in the Greek.
But nobody alive can make me understand an ohm,
And my ignorance of amperes is unique.

And though I'm very proud to know an Englishman was there,
That with all the skill habitually his,
He plays some sort of part in this electrical affair,
For the life of me I can't say what it is.

No matter, for as long as we enjoy their blessed flames
These gentlemen will never be forgot.
Well, even I with reverence can recollect their names;
Thank you, Ampere; thank you, Volta, Ohm, and Watt.

Check and Proofs of the Bernoulli Equation

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THE proof of the Bernoulli equation that has become conventional in textbooks of general physics deals with work and energy in a situation represented in Fig. 1, where we have streamline flow of a liquid, assumed incompressible, in an expanding pipe. The net amount of work done in getting unit volume of the material through two such surfaces as A_1 and A_2 is equated to the gain in its kinetic energy as it goes from one to the other. This gives directly the familiar equation.

The Problem

In the figure the section areas are A_1, A, and A_2, the corresponding velocities of material are U_1, U, and U_2, and the pressures P_1, P, and P_2. The volume between A_1 and A_2 is B. Here material is moving to the right, so that momentum in that direction is entering B through A_1 and leaving B through A_2. It is of interest now to calculate these rates, and the consequent rate of change of momentum within B, and to note the connection of this rate with force, as suggested by Newton's second law of motion.

The time rate at which momentum to the right enters B through A_1, which we will call R_1, is obviously $\rho A_1 U_1^2$, where ρ is the density of the material. Similarly through A_2 momentum is leaving B at the rate R_2 which is $\rho A_2 U_2^2$. The corresponding rates of transfer of mass r_1 and r_2 are $\rho A_1 U_1$ and $\rho A_2 U_2$, which are equal, there being no source or sink of material within B, but actually in that region a steady state. Then since U_1 is greater than U_2, R_1 is greater than R_2, and the rate of gain of momentum within B is R, where

$$R = R_1 - R_2 = \rho A_1 U_1^2 - \rho A_2 U_2^2.$$

Since $r_1 = r_2$, it follows that $U_2^2 = U_1^2 A_1^2 / A_2^2$; therefore, $R = \rho A_1 U_1^2 (1 - A_1/A_2)$, or

$$R = \rho U_1^2 (A_1/A_2)(A_2 - A_1). \tag{1}$$

This rate is positive in the case represented in Fig. 1.

I first encountered the situation just described some 40 years ago. So far as I could have learned from textbooks of general physics, momentum has been accumulating within region B ever since, and must by this time have reached astronomical amounts. No one suggests, in the familiar elementary proofs, that the two velocities at A_1 and A_2, together with the equal rates at which mass passes through these planes, give rise to a continuous increase of momentum, nor that the various forces exerted upon the material within B add up to a horizontal resultant toward the left that is competent to annul that increase. The walls of the tube in the expanding part contribute to this horizontal force.

The Check

We now proceed to check the Bernoulli equation by using the values of pressure P given by that equation in calculating the horizontal force acting on the material in B. If this total force turns out to be directed toward the left, and numerically equal to R, then according to the second law of motion the rate of increase of momentum due to transfer of material is annulled by the horizontal force, the net rate of increase is zero, and we shall save the steady state actually existing in B. In order to calculate the horizontal force exerted by the walls we must integrate PdA over the expanding surface, where dA is the vertical component of an element of that surface, the limits for A being A_1 and A_2.

Let A be any plane vertical section between A_1 and A_2 where velocity and pressure are U and P. The pressure over A_1 gives rise to a force F_1 acting toward the right upon the liquid within B. The pressure exerted by the walls in the expanding part of the tube gives rise to a force F_3, also toward the right. The pressure over A_2 gives rise to a force F_2 toward the left. We proceed to find F, the sum of these forces, taking F_2 as positive, so that $F = F_2 - F_3 - F_1$. The Bernoulli equation gives

$$P_1 + \tfrac{1}{2}\rho U_1^2 = P + \tfrac{1}{2}\rho U^2 = P_2 + \tfrac{1}{2}\rho U_2^2. \tag{2}$$

At the same time

$$A_1 U_1 = A U = A_2 U_2, \tag{3}$$

on account of the constancy of the mass within each part of B, as noted above. We have first $F_1 = P_1A_1$. Then

$$F_2 = P_2A_2 = P_1A_2 + \tfrac{1}{2}\rho U_1^2A_2 - \tfrac{1}{2}\rho U_2^2A_2.$$

Since by Eq. (3) $U_2^2 = U_1^2A_1^2/A_2^2$,

$$\begin{aligned} F_2 &= P_1A_2 + \tfrac{1}{2}\rho U_1^2A_2 - \tfrac{1}{2}\rho U_1^2A_2A_1^2/A_2^2 \\ &= P_1A_2 + \tfrac{1}{2}\rho U_1^2(A_2^2 - A_1^2)/A_2. \end{aligned} \tag{4}$$

Then for the force exerted by the sides of the tube,

$$\begin{aligned} F_3 &= \int_{A=A_1}^{A=A_2} PdA = \int_{A_1}^{A_2} (P_1 + \tfrac{1}{2}\rho U_1^2 - \tfrac{1}{2}\rho U^2)dA \\ &= P_1(A_2 - A_1) + \tfrac{1}{2}\rho U_1^2(A_2 - A_1) \\ &\qquad - \tfrac{1}{2}\rho \int_{A_1}^{A_2} U_1^2(A_1^2/A^2)dA \\ &= (P_1 + \tfrac{1}{2}\rho U_1^2)(A_2 - A_1) \\ &\qquad - \tfrac{1}{2}\rho U_1^2A_1^2 \int_{A_1}^{A_2} dA/A^2 \\ &= (P_1 + \tfrac{1}{2}\rho U_1^2)(A_2 - A_1) \\ &\qquad + \tfrac{1}{2}\rho U_1^2A_1^2(1/A_2 - 1/A_1) \\ &= (P_1 + \tfrac{1}{2}\rho U_1^2)(A_2 - A_1) \\ &\qquad - \tfrac{1}{2}\rho U_1^2(A_2 - A_1)(A_1/A_2) \\ &= \{P_1 + \tfrac{1}{2}\rho U_1^2(A_2 - A_1)/A_2\}(A_2 - A_1). \end{aligned} \tag{5}$$

Therefore

$$\begin{aligned} F &= P_1A_2 + \tfrac{1}{2}\rho U_1^2(A_2^2 - A_1^2)/A_2 - P_1A_1 \\ &\qquad - P_1(A_2 - A_1) - \tfrac{1}{2}\rho U_1^2(A_2 - A_1)^2/A_2 \\ &= \rho U_1^2(A_2 - A_1)(A_1/A_2). \end{aligned} \tag{6}$$

But we showed above that momentum to the right was increasing within B at the rate $\rho U_1^2(A_2 - A_1)(A_1/A_2)$ on account of the motion of material through A_1 and A_2. We have now a total force acting to the left on this material and given by exactly the same expression. Since force equals time rate of change of momentum we have a net rate of change equal to zero. This must be the case if a steady state exists in B. Since at any point fixed within B the velocity does not change, the constancy of the total momentum within B is to be expected. The agreement just found constitutes a check on the Bernoulli equation, unless one prefers to call it a check on the second law of motion.

Energy *Versus* Momentum

Since we have streamline motion within the pipe the velocity of the fluid at any point next to the side has no component normal to the surface, and since we ignore friction the force is normal to the surface. Hence the rate at which work is being done upon the liquid by the surface of the pipe, or upon the pipe by moving liquid, is zero. A student might ask why it is necessary to calculate the contribution made by the walls of the tube to the momentum of the liquid, while it is not necessary to calculate the work done. An elementary problem will clear up this difficulty.

Suppose that a car can be moved without much friction on level steel rails, and that a man, standing on the track, and using a rope and a single pulley, sets the car into motion in such a way that he contributes to the car all its kinetic energy, but only half its momentum. How is the tackle arranged?

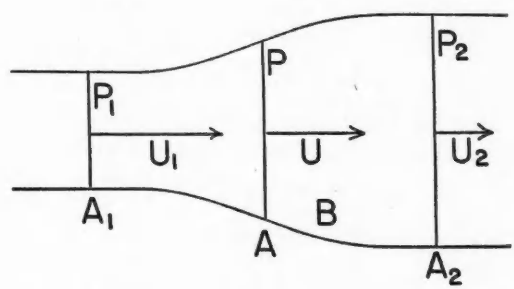

FIG. 1. Streamline flow of an incompressible fluid through a pipe of varying cross section.

Clearly the man must fasten the single pulley to the car, tie one end of the rope to a firm post, and draw the rope through the pulley, so that, when he pulls on the rope, the two parts of it are nearly horizontal and parallel. While the man exerts the force F on the rope and moves the distance s along the track the force exerted on the car is $2F$ and it moves a distance $s/2$, so that the energy expended by the man (Fs) is just that acquired by the car, $2F \times s/2$. However, the momentum gained by the car is $2Ft$, of which the man contributes Ft and the post contributes Ft. The post is, of course, fixed to the earth but the

earth is not, in the ordinary sense, fixed to anything. Here t is the time required for the operation. It is obvious that with a different pulley system the man could compel the post to contribute 9/10, or any other fraction, of the resulting momentum of the car, but he himself would have to supply all the kinetic energy of the car in any case.

A Proof Making Use of Momentum

The foregoing check of the Bernoulli equation is not a proof of that equation, but Newton's second law gives us a very elegant derivation, as follows.

Consider a thin layer of liquid next to surface A in the figure. We have areas, pressures and velocities A, P, and U on the left face, and $A+dA$, $P+dP$, and $U+dU$ on the right. Just as before, the rate of gain of momentum to the right within the layer is R, and

$$R=\rho AU^2-\rho(A+dA)(U+dU)^2.$$

Also, since the mass within the layer does not change, $\rho AU=\rho(A+dA)(U+dU)$. Therefore

$$R=\rho AU\{U-(U+dU)\}=-\rho AUdU. \quad (7)$$

In the case being considered this rate is positive, for dU represents a decrease in velocity. The total force acting to the left on the layer is $(P+dP)(A+dA)-PA-(P+dP/2)dA$, so that, neglecting products of differentials,

$$F=PdA+AdP-PdA=AdP. \quad (8)$$

Hence, as before, $AdP=-\rho AUdU$, or

$$dP+\rho UdU=0. \quad (9)$$

Integrating along a streamline gives $P+\frac{1}{2}\rho U^2=C$, where C is a constant of integration. This is the Bernoulli equation for horizontal flow. Perhaps it is better to evaluate a definite integral from A_1 to A_2, yielding $P_1-P_2=\frac{1}{2}\rho U_2^2-\frac{1}{2}\rho U_1^2$, or, in the usual form,

$$P_1+\tfrac{1}{2}\rho U_1^2=P_2+\tfrac{1}{2}\rho U_2^2. \quad (10)$$

This method amounts to that of general hydrodynamics.[1]

[1] See, for example, H. Lamb, *Hydrodynamics* (Cambridge Univ. Press, Ed. 5, 1924), p. 19.

A Proof Making Use of Power

We have noted that the conventional elementary proof of Bernoulli's equation follows the passage of a particular mass of the liquid into, and out of, the space between fixed planes, as in the figure, and equates gain in kinetic energy of the material transferred to the net amount of work done. A clearer and more elegant proof may be given in terms of instantaneous rates. No time interval is required, no work is done, no material is moved, and one does not have to think of pushing a block of material through the surfaces.

We first write down as before the equal rates at which material enters region B through fixed surface A_1 and leaves B through A_2, giving the equation $A_1U_1=A_2U_2$. Then kinetic energy is entering B through A_1 at the rate $\frac{1}{2}\rho U_1^2A_1U_1$, where $\frac{1}{2}\rho U_1^2$ gives energy per unit volume, and A_1U_1 gives volume per unit time, so that, if energy enters at the rate R_1, then $R_1=\frac{1}{2}\rho A_1U_1^3$. Similarly energy is leaving B through A_2 at the rate R_2, where $R_2=\frac{1}{2}\rho A_2U_2^3$. The net rate of change of kinetic energy is then R, and $R=R_1-R_2=\frac{1}{2}\rho(A_1U_1^3-A_2U_2^3)$. Using the equation describing the movement of material as before we have

$$R=\tfrac{1}{2}\rho A_1U_1(U_1^2-U_2^2). \quad (11)$$

In the figure, U_1 is greater than U_2, hence R is positive and represents a rate of increase of kinetic energy within B. However, just as in the case of momentum, no change of energy can really occur, for we have a steady state within B. We now note that power is an instantaneous rate of doing work, and may, in this case and many others, be represented by force $\times$ velocity. Over the surface A_1 the force exerted *upon* the liquid in B is toward the right, and the velocity U_1 of the liquid is toward the right also. Hence at A_1 work is being done upon this material at the rate $P_1A_1U_1$. This is the rate at which energy is being contributed to the material within B. Over the surface A_2, however, the force exerted *upon* the material within B is toward the left, and at A_2 the velocity U_2 is toward the right. Hence work or energy is not being contributed to this material, but is being subtracted from it, the rate of loss being $P_2A_2U_2$. At A_2, in other words, energy is being passed to the material outside B. Hence work is being done upon B at the net rate R',

where

$$R' = P_1A_1U_1 - P_2A_2U_2.$$

Since $A_1U_1 = A_2U_2$,

$$R' = A_1U_1(P_1 - P_2). \tag{12}$$

This rate must be equal and opposite to R, for since in B there exists a steady state we must have $R+R'=0$, so that

$$\tfrac{1}{2}\rho A_1U_1(U_1^2 - U_2^2) + A_1U_1(P_1 - P_2) = 0,$$

or

$$\tfrac{1}{2}\rho U_1^2 - \tfrac{1}{2}\rho U_2^2 + P_1 - P_2 = 0,$$

and therefore

$$P_1 + \tfrac{1}{2}\rho U_1^2 = P_2 + \tfrac{1}{2}\rho U_2^2, \tag{13}$$

which was to be proved.

In this derivation it is not necessary to take account of the force exerted by the walls of the tube, as we did when using Newton's second law, for since the velocity is parallel to the surface and the force is perpendicular to it, the power is always zero. By the use of instantaneous rates this argument avoids the artificiality of pushing unit volumes of liquid through bounding surfaces, and makes clearer also the instantaneous directions of energy flow. The product of pressure and velocity as used here suggests the Poynting vector of electrodynamics.

I wish to acknowledge helpful discussions of this paper with David B. Sleator and W. W. Sleator, Jr.

The Case for Collective Research

. . . Science is at present organized primarily for war and private profit.

These are not the purposes for which the scientific workers themselves want science to be organized. They want to be free to use their knowledge, in industry and agriculture, in medicine and social science, for peace and not for war, for the benefit of all and not the profit of a few. To secure this we need an organization of science of a new kind: one which sets itself definite ends and considers the most effective means for achieving them. Quite apart from the size of science, the increasing complexity and interdependence of its different parts —the use, for instance, of nuclear fission to produce tools for biochemical research—show that the whole system needs to be linked together so that every section can profit as soon as possible from the results of any of the others. We can no longer, as we could in past ages, leave the organization of science to chance or to the personal relations of individual scientists.

It does not, of course, follow that any kind of organization would be appropriate for science. The mere task of finding the kind of organization needed for science is itself a scientific problem. For science differs from all other human enterprises by dealing, not with the known and the regular, but with the unknown and unexpected.—J. D. BERNAL, *Bulletin of the Atomic Scientists* **5**, 17 (1949).

The Case for Individualism

Scientists must be Independent.—The pursuit of science can be organized, therefore, in no other manner than by granting complete independence to all mature scientists. They will then distribute themselves over the whole field of possible discoveries, each applying his own ability to the task that appears most profitable to him. Thus, as many trails as possible will be covered, and science will penetrate most rapidly in every direction toward that kind of hidden knowledge which is unsuspected by all but its discoverer, the kind of new knowledge on which the progress of science truly depends. The function of public authorities is not to plan research, but only to provide opportunities for its pursuit. All that they have to do is to provide facilities for every good scientist to follow his own interests in science. To do less is to neglect the progress of science; to do more is to waste public money. Such principles have in fact essentially guided all well-conducted universities throughout the modern age.—MICHAEL POLANYI, *Bulletin of the Atomic Scientists* **5**, 20 (1949).

Graph Papers as Instruments of Calculation

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General Principles

THE techniques described in this article have arisen from the author's interest in graphical and mechanical methods of computation. While some may be original, it is almost certain that many of them have been described and used before; in any event, the author makes no claim either of originality or of exhaustive treatment.

The purposes of this article are, primarily:

(a) to transmit to others this material, which the author has found useful in simplifying engineering calculations;

(b) to inaugurate a literature of methods of calculation involving graph papers;

(c) to stimulate others in publishing similar methods of their own or in devising related methods; and

(d) to provide, for those unacquainted with nomography, an introduction employing a tool already familiar to them—the common graph papers.

The methods described here are not to be considered as having the accuracy of slide-rule calculations. They correspond rather to nomographs in this respect, although graph papers are generally machine-ruled and, consequently, more accurate than hand-ruled nomographs. The accuracy of any one method is largely determined by the particular operations involved and the type of graph paper used. For rough calculations, such as estimates and checks, and for calculations for which the original data contain errors up to one percent these methods are frequently sufficiently accurate. Their claim to usefulness lies largely in (i) the specific nature of the routines as opposed to the general nature of slide-rule calculations, (ii) the fact that in most of the calculations the position of the decimal point is automatically determined, and (iii) the ready availability and low cost of a wide variety of graph papers.

The types of graph papers to be included in the present article are:[1] hyperbolic (or reciprocal ruled), linear (or cross section), semilogarithmic (or ratio), and logarithmic (or log-log). The method of solution generally employed is that of a straight index line crossing the scales of the graph paper. For proof of each method, the corresponding equation of the straight line is employed.

The three common forms of the equation of the straight line in analytic geometry as applied to nomographic constructions are: (a) The two-point form,

$$y_1(x_2-x_3)-y_2(x_1-x_3)+y_3(x_1-x_2)=0, \quad (1)$$

in which (x_1, y_1), (x_2, y_2), and (x_3, y_3) represent three points located on the straight line. The coordinates of any one of these points may be considered as the variables in Eq. (1), the other two points being sufficient to determine the line; (b) the intercept form,

$$x/a+y/b=1, \quad (2)$$

in which x and y are coordinates of a point intersected by the index line and a and b are the X and Y intercepts; (c) the point-slope form,

$$y=mx+b, \quad (3)$$

in which x, y, and b have the same meanings as in Eq. (2), while m is the slope of the index line.

As employed in the proofs of certain relationships in this article, the points appearing in Eqs. (1), (2), and (3) represent not isolated points but sets of points intersected on three scales by a straight line, as shown in Fig. 1.

Hyperbolic Paper

Hyperbolic paper has the vertical axes subdivided linearly and the horizontal axes subdivided by an hyperbolic scale defined by the function, $x=s(p-1)/p$, where s is the length of this scale and x is the distance from the origin to the scale point marked p. For example, if $p=2$, then the point 2 on this scale will lie at a distance $x=s(2-1)/2$ or $s/2$ from the origin of the scale. For the following applications, the length s may be designated as unity without loss of generality; whence, $x=(p-1)/p$. The divisions of the hori-

[1] The author wishes to acknowledge the kindness of the Keuffel and Esser Company, Hoboken, New Jersey, which generously furnished for this study supplies of the following graph papers: hyperbolic, 359–25; linear 10/10/in., 359–5; semilog 10/in.-1 cycle, 359–51; semilog 10/in.-2 cycle, 359–61; semilog 10/in.-5 cycle, 359–91; log-log 3×3 cycle, 359–120; and log-log 2×2 cycle, 359–110.

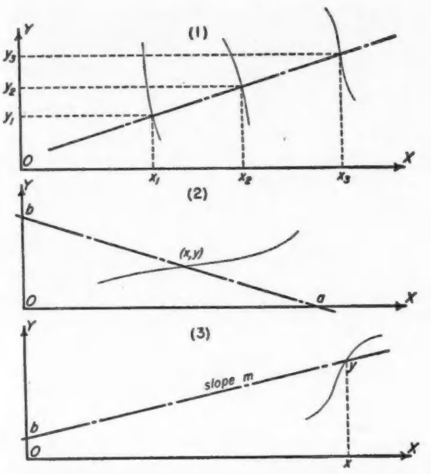

FIG. 1. Common forms of the equation of the straight line: (1) the three-point form; (2) the intercept form; and (3) the point-slope form.

zontal axes are marked in terms of p, rather than x. It is from this subdivision of the X axis that this type of graph paper derives its name. The alternate name, reciprocal, arises from the fact that the ruling of the X axis in the reverse direction is $x'=s(1-x)=s/p$, or for the case where $s=1$, $x'=1/p$.

It might appear odd at first that this paper, rather than linear paper, should have been chosen as a starting point for this discussion. The three primary reasons for this choice are (i) the hyperbolic ruling is closely associated with nomography,[2] (ii) the relationships of hyperbolic paper carry over into the applications of linear and logarithmic papers, and (iii) linear paper is in reality a special type of hyperbolic paper. The author has demonstrated[3] the use of hyperbolic coordinates of a more general type in nomography, namely, coordinates whose X axis is subdivided, $x=sp/(p+r)$, where r is a variable which may assume any one of a number of convenient constant values. For the special case, $s=\infty$, $r=\infty$;

$$x=\lim_{\substack{r\to\infty\\ s\to\infty}}\frac{sp}{p+r}=\lim_{\substack{r\to\infty\\ s\to\infty}}\frac{p}{\dfrac{p}{s}+\dfrac{r}{s}}=p;$$

hence, in the limit, x becomes linear. It is impossible in this brief article to exploit the versatility of this paper fully, but the examples given will serve to illustrate some of its uses, and will possibly suggest others.

1. Averages. Line (*a*) in Fig. 2 shows averages of two numbers, $N_1=56$ and $N_2=82$. For the case in which these numbers are of equal weight ($w_1=w_2=1$), this average, 69, is found at the intersection of the index line with the middle vertical line, $p=2$ (or $p=w_1+w_2=2$). For the case in which N_2 has three times the weight of N_1 (or $w_2=3$, $w_1=1$), the weighted average ($A=75.5$) is found at the intersection of the index line with the vertical three-fourths of the distance from N_1 to N_2, $p=4$ (or $p=w_1+w_2=4$). In fact, in every case in which one number has a weighting of 1, it may be located on the left-hand axis, the other number on the right-hand axis, and the average on the vertical whose value is equal to the sum of the weightings.

For cases involving averages of two numbers, neither weighting of which is unity, the first number is located on the vertical whose value is equal to its weighting. Line (*b*) of Fig. 2 shows the average of $N_1=49$ with weighting $w_1=1.8$ and $N_2=35$ with weighting $w_2=2.6$. The average $A=40.8$ is located at the intersection of the index line with the vertical at $p=1.8+2.6=4.4$.

This last case may be extended to the average of a number of weighted terms, N_1w_1, N_2w_2, N_3w_3, $\cdots$ etc., for, when the average of the first two is found, it will lie on the vertical at $p=w_1+w_2$, which is the starting point for averaging

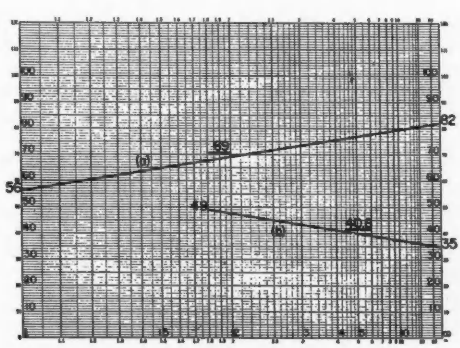

FIG. 2. Averages. (*a*) The arithmetic mean of 56 and 82 is 69. (*b*) A weighted average: the average of 49 with weighting 1.8 and 35 with weighting 2.6 is 40.8.

[2] See W. H. Burrows, *J. Eng. Ed.* **36**, 361 (1946), and *Ind. Eng. Chem.* **38**, 472 (1946).
[3] *Ind. Eng. Chem.* **38**, 472 (1946).

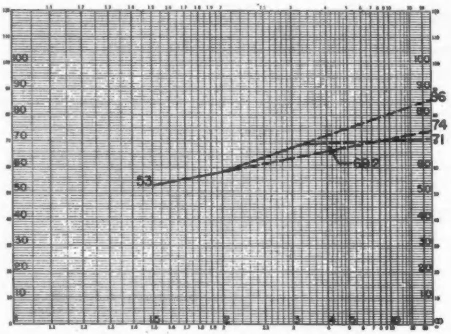

FIG. 3. Weighted average of four terms. The average of 53 with weighting 1.5, 74 with weighting 0.5, 86 with weighting 1.2, and 71 with weighting 4.0 is 69.2.

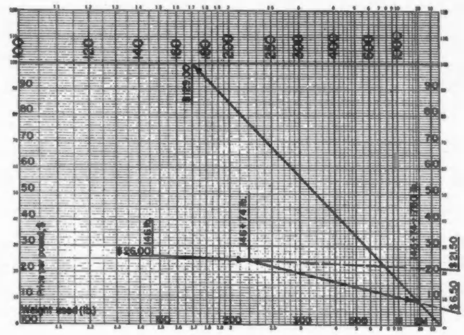

FIG. 4. A weighted sum. Cost per pound and total cost of an alloy.

this first average with the third term, N_3w_3. The second average will then lie at the proper point for averaging the fourth term, and so forth for each additional term. All terms, except the first, are located on the right-hand axis. The first is located at $p=w_1$. Figure 3 shows the average of the following terms:

NUMBER	WEIGHTING	TOTAL WEIGHTING
53	1.5	1.5
74	0.5	2.0
86	1.2	3.2
71	0.8	4.0

Average = 69.2.

Proof of the above relationships is obtained by substituting in Eq. (1): $x_1=(p_1-1)/p_1$, $x_2=(p_2-1)/p_2$, $x_3=1(p_3=\infty)$, $y_1=y_1$, $y_2=y_2$, and $y_3=b$, whence:

$$y_2=\frac{p_1y_1+(p_2-p_1)b}{p_2}. \tag{4}$$

If the following substitutions are then made in Eq. (4): $y_1=N_1$, $b=N_2$, $p_1=w_1$, and $p_2=w_1+w_2$, then

$$y=\frac{w_1N_1+w_2N_2}{w_1+w_2}=A,$$

where A is the average of the weighted terms, w_1N_1 and w_2N_2.

2. *Sums.* Weighted sums may be obtained simultaneously with weighted averages by projecting the weighted average through the point $(p=\infty, y=0)$ onto the Y axis or onto a horizontal projection of the Y axis. Figure 4 shows (*a*) the average cost per one hundred pounds and (*b*) the total cost of an alloy whose composition is:

METAL	COST PER 100 LB	WEIGHT USED (LB)
Tin	\$26.00	1.46×10^2
Antimony	21.50	0.74×10^2
Lead	6.50	17.80×10^2
Cost per 100 pounds: \$8.46		Total cost: \$169.20

The horizontal scale for sums is a projection of the left-hand axis, using the point $(p=\infty, y=0)$ as a projection point. This same point is used in projecting the average onto this scale to obtain the sum; hence the index line (*b*) which intersects this scale for sums would, if extended, intersect the left-hand axis in a point of equal value. The equation of this line may be obtained by substituting in Eq. (2) the values $x=(p-1)/p$, $a=1$, and $y=y$; whence

$$b=py. \tag{5}$$

If in Eq. (5) the further substitutions are made: $y=A$ and $p=w_1+w_2+\cdots$, then $b=A(w_1+w_2+\cdots)=S$, where $S=N_1w_1+N_2w_2+\cdots$. Therefore, S is the desired sum.

3. *Products and powers.* The method described in the preceding section for obtaining sums suggests a convenient method for products. Indeed. Eq. (5), $b=py$ is the product so obtained, Figure 5(*a*) shows the product $40\times8=320$ obtained by this method, using the projected horizontal scale for products. An analogous method for squares and square roots may be used, for if in Eq. (5), $p=y$, then $b=y^2$. Figure 5(*b*) shows that $15^2=225$, or, by a reverse process, $\sqrt{225}=15$. This method might be altered slightly for ob-

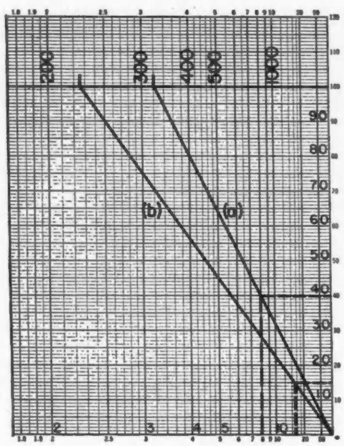

FIG. 5. Products and powers. (*a*) The product of $40\times 8=320$. (*b*) The square of $15=225$.

taining higher powers and roots, as by scaling $p=y^2$, whence $b=y^3$.

A special case which might have application in certain formulas is the product obtained by using some point other than zero on the right-hand scale as origin of the index line. In this case the projected b-scale cannot be used. The equation of a line intersecting the left-hand axis at b, the right-hand axis at a, and passing through the point (p, y) is obtained by substituting in Eq. (1) the following values: $x_1=0$, $x_2=(p-1)/p$, $x_3=1$, $y_1=b$, $y_2=y$, $y_3=a$; whence

$$p=\frac{b-a}{y-a}. \tag{6}$$

This formula is presented at this point largely because of its application to the following section.

4. *Quadratic equations.* The quadratic equation $y^2+Ay+B=0$ may be solved by applying the following substitutions to Eq. (6): Let $a=-A$, $b=-(A+B)$, and $p=y$. Thus, to solve a quadratic equation on hyperbolic paper, locate the value $-A$ on the right-hand axis and the value $-(A+B)$ on the left-hand axis. Lay a straightedge between these points; the value of p at which the straightedge intersects equal values of p and y is a root of the equation. For instance, Fig. 6 shows the two roots of the equation, $y^2-5y+6=0$, to be $+2$ and $+3$.

If only one root appears, the other is a negative root, or if no root appears by the first substitution, both roots are negative. Negative roots are located by the substitutions: $a=-A$, $b=-(A-B)$, and $p=-y$, which lead to the same quadratic equation, $y^2+Ay+B=0$. Thus, to solve for the negative roots of the quadratic, lay the straightedge between $a=-A$ and $b=-(A-B)$. The value of $p=-y$ intersected by the straightedge is a negative root of the equation. Figure 7 shows the two roots of the equation $y^2+y-6=0$ to be $+2$ and -3.

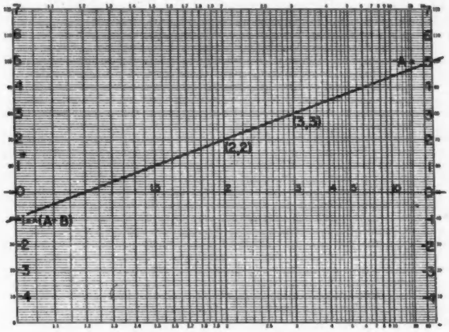

FIG. 6. Solution of a quadratic equation, $y^2+Ay+B=0$. The roots of $y^2-5y+6=0$ are $+2$ and $+3$.

This method may also be extended to the reduced forms of the cubic equation by making similar substitutions for p and y. To solve the equation, $y^3+Ay^2+B=0$, substitute $p=y^2$; and for $p^3+Ap+B=0$, put $y=p^2$.

The substitutions given above for the positive and negative roots of quadratics are satisfactory only for equations whose roots have absolute values within a range of approximately 1 to 10. Below this range the roots lie off the hyperbolic scale; above this range, accuracy is lost because of the crowding of scale points in the higher values of p. This difficulty is circumvented by the following device.

First, the scale points of the hyperbolic scale are multiplied by some factor, r, so that the desired range of y now lies in the lower or middle position of the scale. For a range of 10 to 100, the factor $r=10$ might be employed, whence the hyperbolic scale points would be 10, 20, 30, etc., instead of 1, 2, 3, etc.

Second, the substitutions made for solution of the quadratic would be, for positive roots, $a=-A$, $b=-(A+B/r)$, and $p=y$; for negative

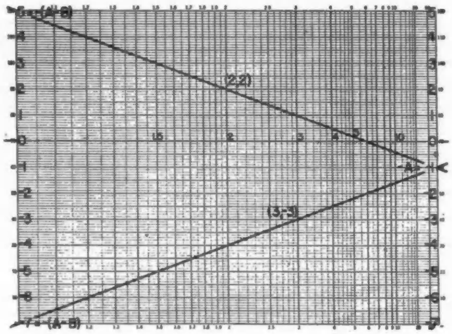

FIG. 7. Negative roots of a quadratic equation. The roots of $y^2+y-6=0$ are $+2$ and -3.

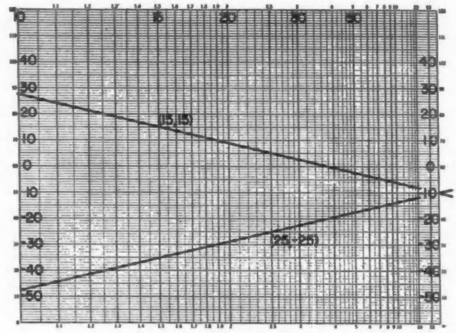

FIG. 8. Equations with large coefficients and roots. The roots of $y^2+10y-375=0$ are $+15$ and -25.

roots, $a=-A$, $b=-(A-B/r)$, and $p=-y$, where r is the factor employed in modifying the hyperbolic scale. Solution of the equation is accomplished as previously described, except for the factor r in the expression for b. Figure 8 shows the roots of the equation $y^2+10y-375=0$ to be $+15$ and -25. The values of a and b employed in this solution are $a=-10$, $b=-(10-37.5)$ for positive roots, and $b=-(10+37.5)$ for negative roots.

5. *Reciprocal sums.* Reciprocal sums, such as occur in the lens formula,

$$1/s_f=1/s_i+1/s_o \qquad (7)$$

expressing the relationship between the focal length s_f, image distance s_i, and object distance s_o, of a convex lens may be solved by means of a straight line intersecting points on parallel hyperbolic scales. The scale points of the middle scale are renumbered so that each point has half the value of the corresponding point on the exterior scales. Figure 9 shows solution of the relationship, $1/1.35+1/3.5=1/0.975$, the first two numbers located on the exterior scale, the last on the middle scale. Proof of the relationship thus established is obtained by substituting in Eq. (1): $x_1=0$, $x_2=\frac{1}{2}$, $x_3=1$, $y_1=1/s_o$, $y_2=1/2s_f$, $y_3=1/s_i$. With these substitutions, the origins of the reciprocal s-scales become the points marked ∞ on the hyperbolic scales. These substitutions transform Eq. (1) into Eq. (7).

This application illustrates the reciprocal nature of the hyperbolic scale. Used with the point $p=1$ as origin, the scale represents the hyperbola, $x=(p-1)/p$. If the origin is taken at the point $p=\infty$ and the scale is read in the reverse direction, it represents the reciprocal, $x'=1/p$. Since the ranges of the hyperbolic scales may be changed by simply multiplying all scale points by a suitable factor (as 0.01, 0.1, 10, etc.), this method is easily applied to a wide range of calculations.

Linear Paper

A number of the calculations which may be performed with hyperbolic graph paper may also be performed with linear paper. There are, however, certain advantages in the use of linear paper for some of the simpler calculations, including the greater familiarity with and availability of this type of paper. A considerable advantage in the use of this paper is the simplicity of the mathematical relationships, these being simply those given in Eqs. (1), (2), and (3) for the straight line. Similar simplicity arises also in calculations employing linear paper with circles or other curves, although few calculations of this type are considered in this paper.

6. *Averages.* Simple averages and certain weighted averages are easily calculated on linear graph papers employing the principles already considered. These will not be repeated at this point. More involved averages and multiple averages are more difficult to calculate on this paper than on hyperbolic paper, and will not be discussed for that reason.

7. *Proportionality.* Perhaps the most interesting and versatile type of calculation readily performed with linear paper is that of proportionality. Two vertical axes of the paper are

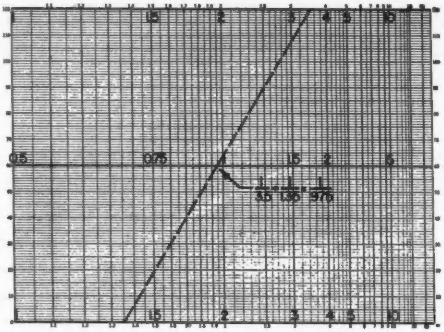

FIG. 9. Reciprocal sums. Solution of the lens formula, $1/s_o+1/s_i=1/s_f$. $s_o=3.5$, $s_i=1.35$, $s_f=0.975$.

scaled over the ranges to be employed, one reading up, the other, down. The zero points of the two scales are then connected by a straight line (Fig. 10). To solve for d in the equation $a/b=c/d$, a is located on one scale, b on the other, and a straightedge is passed through both points. The intersection of the straightedge with the diagonal is marked, and the straightedge is rotated about this point until it intersects c on the first scale. In this position the straightedge intersects the second scale at d. Proof may be readily established geometrically by the similarity of triangles cut off by the straightedge or by the following substitutions in Eq. (1): $y_1=a$, $y_2=0$, $y_3=-b$, $x_1=0$, $x_2=x$, $x_3=1$; whence $a(x-1)+bx=0$, or $x=a/(a+b)$. Likewise, $x=c/(c+d)$.

By eliminating x between these two expressions,

$$a/(a+b)=c/(c+d)$$

or

$$a/b=c/d. \tag{8}$$

Figure 10 shows the proportion $7/10=28/40$.

A special case of proportionality is the geometric mean, $w^2=uv$. Either u or v is easily calculated by the above method by setting $a=u$, $b=c=w$, and $d=v$. The value of w is not impossible to find by a reverse process, but the method discussed in Sec. *9* is much quicker, as is that in Sec. *11*.

Another special case is the evaluation of powers of numbers. For instance, $w^2=u$ may be found by substituting $a=u$, $b=c=w$, and $d=1$ in Eq. (8). In like manner, w^2 having been calculated, w^3 may be obtained by setting $b=w$, $c=w^2$, etc.

A somewhat different form of proportionality, also based on similarity of triangles, is that shown in Fig. 11. This form is especially useful in converting grades from some odd basis to a basis of 100. Suppose the total of points which may be earned on a quiz is 72. It is desirable to convert the points earned (numerical grades) to a basis of 100 (percentage grades) for sake of comparison. The method shown in Fig. 11 immediately indicates that a grade of 65 points out of 72 equals a grade of 90 (actually 90.3) percent. Obviously, the conversion could as easily be made to any other basis as well as to the basis of 100.

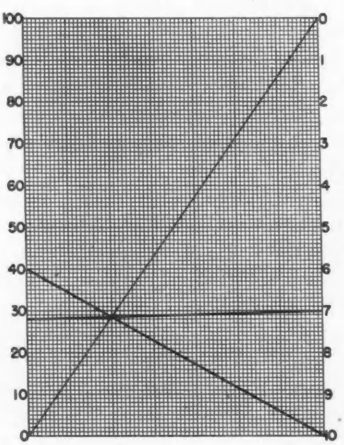

FIG. 10. Proportionality. $7/10=28/40$.

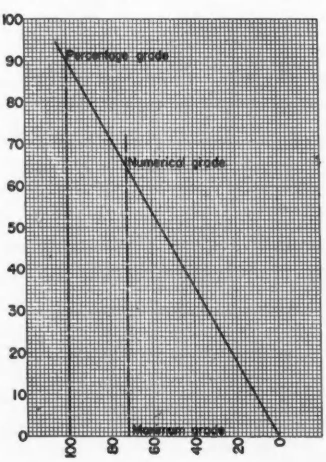

FIG. 11. Percentage grades: 65 points out of a maximum of 72 is equal to a percentage grade of 90.3.

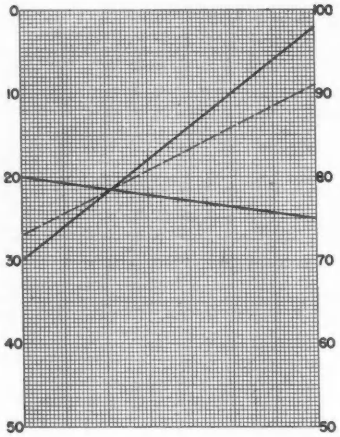

FIG. 12. Linear interpolation. Evaluation of log1.9227 = 0.28391.

8. Interpolation. Linear interpolation between successive values of a function in a table is accomplished by first establishing a proportionality between a given interval of the independent variable on one scale and the corresponding interval of the dependent variable on a parallel scale. The proportionality thus established is then employed in locating the value of the dependent variable corresponding to an intermediate value of the independent variable.

For simplicity and compactness, it is desirable that the lines establishing the initial ratio intersect at some point between the two scales. The following rules establish this condition: (1) If the dependent variable *increases* with increasing value of the independent variable, the scales representing these variables should read in *opposite sense.* (2) If the dependent variable *decreases* with increasing values of the independent variable, the scales should read in the *same sense.*

Figure 12 shows interpolation between successive values of log*n*. The *interval* rather than the actual number is important; hence, only the last two digits of each number are employed. In this figure, it is desired to find log1.9227. The table of common logarithms gives the following values:

n	log*n*
1.9220	0.28375
(1.9227	——)
1.9230	0.28398.

Since log*n* increases as *n* increases, scales of opposite sense are employed. The interval 20–30 is located on one scale; the interval 75–98 on the other. Using the ratio thus established, the value corresponding to 27 on the first scale is found to be 91.1. Therefore, log1.9227 = 0.28391. Since logarithms are not linear functions, linear interpolation will not be entirely accurate. The value of this logarithm as listed in six-place tables is 0.2839112; hence the accuracy is sufficient to lend value to the method (provided a sufficient number of such interpolations are required to warrant use of a graphical method).

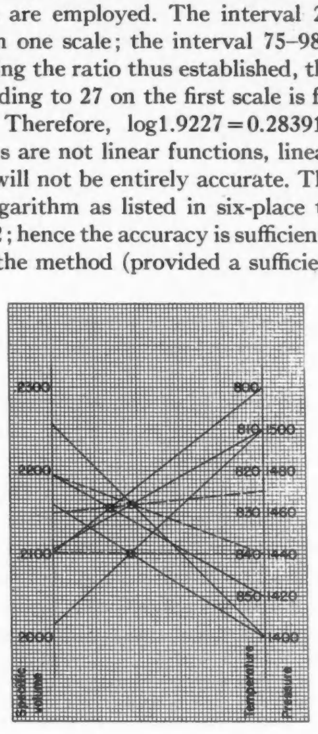

FIG. 13. Double interpolation. Calculation of specific volume of carbon dioxide at 825°F and 1440 lb in.$^{-2}$ pressure.

Double interpolation of a variable which is a function of two independent variables is accomplished in similar fashion. Two interpolations (for successive columns) are made between successive values of one independent variable. A third interpolation is then made between the values thus obtained with respect to the second independent variable. Figure 13 shows the method applied to the value of the specific volume of CO_2 at 825°F and 1440 lb in.$^{-2}$ pressure. Values given by a table are:[4]

P (lb in.$^{-2}$)	800°F	850°F
1400	0.2160	0.2257
1500	0.2014	0.2106

The desired value of *v* is found to be 0.2148.

[4] Sweigert, Weber, and Allen, "Thermodynamic properties of gases," *Ind. Eng. Chem.* **38**, 196 (1946).

9. Geometric mean. One of the theorems of elementary geometry states that a perpendicular from the diameter of a circle to its circumference is a mean proportional between the segments of the diameter adjoining the perpendicular. The proof lies in the similarity of triangles, as shown in Fig. 14, wherein $\angle A = \angle A'$, $\angle B = \angle B'$, and $\angle C = \angle C'$. In applying this theorem to the calculation of the geometric mean with linear graph paper, it is unnecessary to draw the circle if a further theorem on circles be applied; namely, an angle inscribed in a semicircle is always a right angle. Thus, a right-angle triangle may be employed in lieu of the compass for this solution. Figure 15 shows the calculation of the geometric mean between the numbers 14 and 30, that is, of n in the equation $14/n = n/30$. The result is 20.5. By setting one of the segments of the diameter equal to one, this method may also be employed for calculating squares and square roots.

10. Reciprocal sums. In Sec. *5* a method was described for solving reciprocal sums, such as

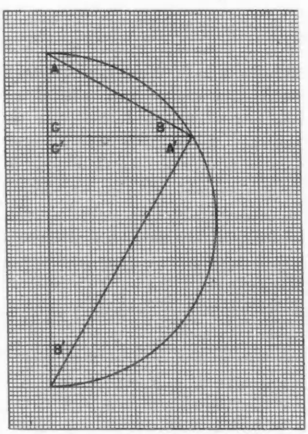

FIG. 14. Similar triangles formed by the diameter of a circle and a line perpendicular to it.

those occurring in the lens formula. For sums of more than two terms, the following method is possibly more satisfactory.

An auxiliary axis of slope $m = 1$ is drawn through the origin of the linear graph paper. Regardless of the moduli (length of unit subdivision) of the X and Y axes, the equation of this auxiliary axis is

$$y = x. \tag{9}$$

An index line through points on the X and Y axes will intersect the auxiliary axis at some point (x, y) whose position is given by Eq. (2). But on the auxiliary axis, $x = y$; hence

$$x/a + x/b = 1,$$

or

$$1/x = 1/a + 1/b. \tag{10}$$

That is, the abscissa (or, by similar proof, the ordinate) of the point intersected on the auxiliary axis is the reciprocal sum of the intercepts of the index line. This abscissa may then be taken as the x intercept of a second index line, the y intercept being a third term of the sum. In this manner, reciprocal sums of any number of terms may be evaluated. Figure 16 shows a solution of the formula for parallel resistors:

$$1/R = (1/r_1) + (1/r_2) + (1/r_3) + \cdots.$$

A second advantage of this method over that of Sec. *5* is its application to sums of positive and negative terms. The axes are chosen such that their negative portions, as well as positive, lie on the paper; the method of solution is then identical with the above method.

Semilogarithmic Paper

The horizontal ruling of semilogarithmic paper is linear, while the vertical ruling is logarithmic. A number of its applications involve the intersection by a straight index line of three equally

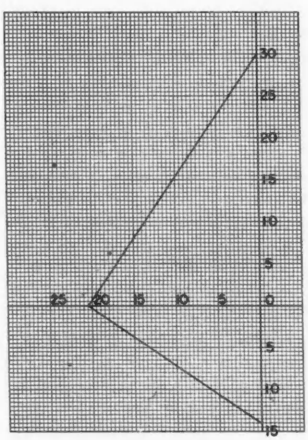

FIG. 15. Geometric mean. The geometric mean of 14 and 30 is 20.5.

spaced vertical scales. The equation of the index line in terms of its points of intersection with the vertical scales may be obtained by substituting in Eq. (1): $x_1=0, x_2=\frac{1}{2}, x_3=1, y_1=\log a, y_2=\log Y, y_3=\log b$. With these substitutions, Eq. (1) becomes

$$-\tfrac{1}{2}\log a+\log Y-\tfrac{1}{2}\log b=0;$$

whence

$$2\log Y=\log a+\log b,$$

or

$$Y^2=ab. \tag{11}$$

11. Squares and square roots. If, in Eq. (11), $b=1$, then $Y^2=a$. That is, if the index line passes through the origin of one of the outer scales, it will also intersect on the other outer scale a value which is the square of the value intersected on the middle scale.

For example, in Fig. 17 the line (A) shows that the square root of 2000 (scale A) is 44.7 (scale Y), as indicated by an index line passing through the value 1 on scale B. Conversely, $44.7^2=2000$.

12. Geometric mean. The equation of the geometric mean, $a/Y=Y/b$, may be written $Y^2=ab$, which is Eq. (11). It is apparent, then, that the value intersected on the middle scale is the geometric mean of the two values intersected on the exterior scale. For example, in Fig. 17 the line (B) shows that the geometric mean of 250 (scale A) and 8 (scale B) is 44.7 (scale Y).

The use of two index lines intersecting at the middle scale increases the range of usefulness of

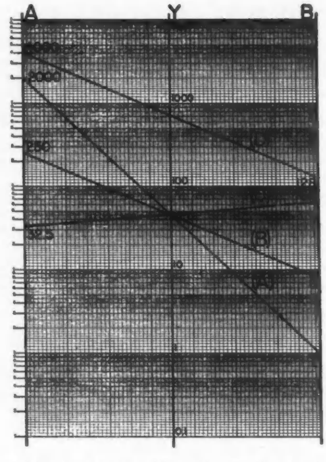

FIG. 17. Calculations made with semilogarithmic paper. (a) $\sqrt{2000}=44.7$. (b) The geometric mean of 250 and 8.0 is 44.7. (a) and (b) The product $8\times250=2000$. (b) and (c) The proportionality $32.5/250=8/64$. (b) and (d) The proportionality, $8/250=128/4000$.

this paper. The equation resulting from this device may be arrived at by making two separate substitutions in Eq. (1) to obtain the equations: $2\log Y=\log a+\log b$, and $2\log Y'=\log c+\log d$. If $Y=Y'$, then $\log a+\log b=\log c+\log d$, or

$$ab=cd. \tag{12}$$

13. Simple products. Simple multiplication employs two index lines, one of which passes through the origin of one of the exterior scales. This same index line intersects on the other exterior scale a value which is the product of the two terms intersected on the exterior scale by the other index line. In short, if in Eq. (12), $d=1$, then

$$ab=c.$$

The lines (A), (B) in Fig. 17 show a practical example of this device. The product obtained is $8\times250=2000$. One index line (B) passes through 8 and 250; the other (A) through 1 and 2000. The two intersect at the middle scale.

14. Combined multiplication and division. From Eq. (12), $(a\times b)/d=c$. Thus, if the second index line passes through the value d, rather than 1, the product obtained will be divided by d. For example, in Fig. 17, the lines (B), (C) show that the value of the product $(8\times250)/64$ is 32.5. The first index line passes through 8 and 250. The

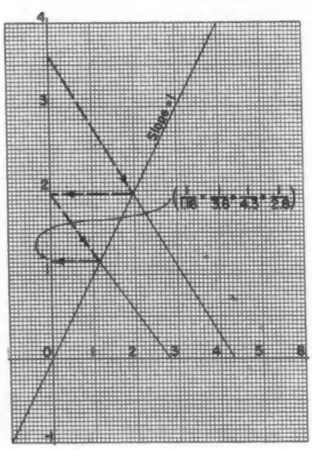

FIG. 16. Reciprocal sums. Solution of the parallel resistor formula, $1/r_1+1/r_2+1/r_3=1/R$.

second, passing through 64 on one exterior scale, intersects 32.5 on the other.

This device may be extended to multiple products, such as are found in calculations involving ideal gases. For example we may have to evaluate $V_2 = V_1T_2P_1/T_1P_2$, where the symbols have the usual significance. First, the product $V' = V_1T_2/T_1$ is obtained; then $V_2 = V'P_1/P_2$.

15. Proportionality. The foregoing is merely a form of proportionality, as may be observed by dividing Eq. (12) by bc:

$$a/c = d/b.$$

Thus, in Fig. 17, the lines (*B*), (*C*) show the proportionality $32.5/250 = 8/64$.

Another form of proportionality represented by parallel lines is illustrated in Fig. 17 by (*B*), (*D*), which are drawn for the case

$$8/250 = 128/4000.$$

The proof of the general case is obtained by substituting in Eq. (3) the values, $y = \log A$, $x = 1$, $b = \log B$, and $m = \log M$; whence $\log A = \log M + \log B$, or

$$A = M \times B. \tag{13}$$

By similar substitution,

$$C = M' \times D. \tag{13'}$$

Division of Eq. (13) by Eq. (13′) gives the proportionality,

$$A/C = M \times B/M' \times D.$$

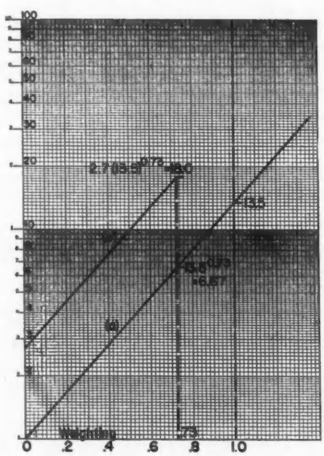

FIG. 18. Fractional powers. (*a*) $13.5^{0.73} = 6.67$. (*b*) $2.7 \times 13.5^{0.73} = 18.0$.

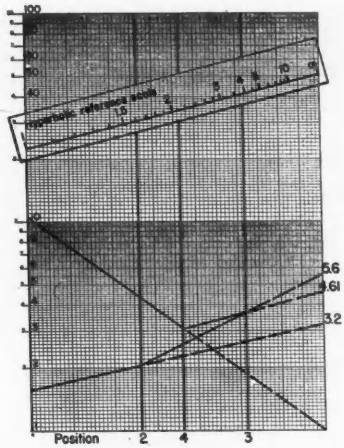

FIG. 19. Device for solution of the formula $x = 1.57a^{0.61}b^{2.2}/c^{1.7}$.

If $M = M'$, then $A/C = B/D$, which is the proportionality desired. But if $M = M'$, then $m = m'$; that is, the slopes of the two index lines are equal; hence, the lines are parallel.

16. Fractional powers. A somewhat different set of substitutions, $y = \log Y$, $m = \log M$, and $b = 0$ converts Eq. (3) into the form $\log Y = x \log M$, or

$$Y = M^x. \tag{14}$$

For the case, $x = 1$, $\log Y = \log M$, while if $x = 0$, $\log Y = 0$, or $Y = 1$. Hence, the index line will pass through the origin ($x_1 = 0$, $Y_1 = 1$) and the point ($x_2 = 1$, $Y_2 = M$). The value of $Y = M^x$ will then be located at the point on this index line whose abscissa is x.

Figure 18 illustrates the case $Y = 13.5^{0.73} = 6.67$. If, in the above substitutions, $b = \log B$, then $\log Y = \log B + x \log M$, or

$$Y = BM^x. \tag{15}$$

It is thus possible, by using an index line of the same slope, but with the y-intercept $b = \log B$, to obtain a fractional power and product. This is illustrated by (*b*) in Fig. 18, which shows the product

$$Y = 2.7(13.5)^{0.73} = 18.0.$$

Additional methods of evaluating these and more extensive products are described by McMillen[5] and Burrows.[6] Both methods employ

[5] McMillen, *Ind. Eng. Chem.* **30**, 71 (1938).
[6] Burrows, *Ind. Eng. Chem.* **38**, 586 (1946).

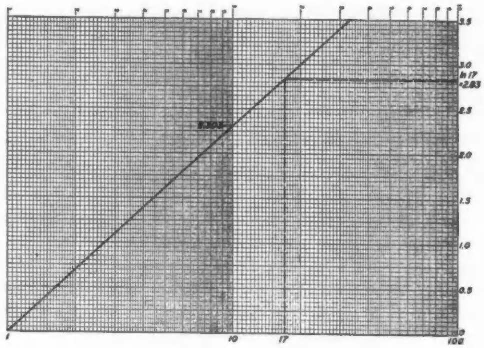

FIG. 20. Natural and common logarithms. Calculation of the natural logarithm, ln 17 = 2.83.

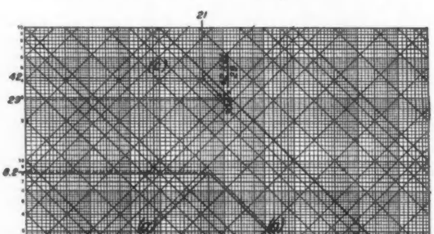

FIG. 21. Simple products. (*a*) The product, $23\times 8.2 = 188.6$ and the quotient, $23/8.2 = 2.8$. (*b*) The product, $23\times 8.2\times 0.52 = 98.1$.

parallel logarithmic scales interspersed by a family of parallel lines spaced hyperbolically. The latter method is exactly analogous to the method for sums described in Sec. *2*. These methods may be adapted to graph paper calculations by combining logarithmic paper and hyperbolic paper, or by ruling vertical hyperbolically spaced lines on semilogarithmic paper. In view of these previously published papers, the method will not be described here for the general case.

It is simple, however, to apply the method of Sec. *2* to products such as the formula illustrated in Fig. 19. The equation is $x = 1.57a^{0.61}b^{2.2}/c^{1.7}$, which may be written in logarithmic form as

$$\log x = \log 1.57 + 0.61 \log a + 2.2 \log b - 1.7 \log c.$$

This logarithmic form is then treated as a weighted sum, following the method of Sec. *2*. The positions of the vertical pivot-axes are determined from an hyperbolic scale laid between the vertical logarithmic scales, the positions being determined from the coefficients as follows:

Position 1 = 1st coefficient = 1.00;
Position 2 = 1st coefficient + 2nd coefficient = 1.00 + 0.61 = 1.61;
Position 3 = sum of the first three coefficients = 1.00 + 0.61 + 2.2 = 3.81;
Position 4 = sum of the four coefficients = 1.00 + 0.61 + 2.2 − 1.7 = 2.11.

The constant term, 1.57, and the final product are located on the left-hand scale. All other terms are located on the right-hand scale, while the pivot moves along the index line from the left-hand scale to Position 1, then to each of the other positions in order.

In Fig. 19, $a = 3.2$, $b = 5.6$, $c = 4.61$, and

$$x = 1.57\times 3.2^{0.61}\times 5.6^{2.2}/4.61^{1.7} = 10.5.$$

17. Natural and common logarithms. The relationship between natural logarithms and common logarithms is

$$\ln a/\ln 10 = \log a/\log 10. \qquad (16)$$

In practice, this relationship is simplified by substituting for ln10 its numerical value $m = 2.303$,

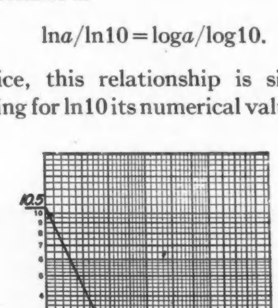

FIG. 22. Products with fractional powers. The product $1.57\times 3.2^{0.61}\times 5.6^{2.2}/4.61^{1.7} = 10.5$.

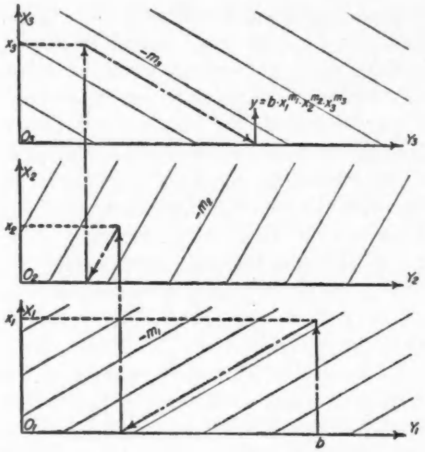

FIG. 23. Method of evaluating products with fractional powers; m_1, m_2, and m_3 are the values of the exponents, and b is the constant coefficient.

and for log10 its numerical value 1. Thus,

$$\ln a = 2.303 \log a. \qquad (16')$$

For the present purpose, the relationship of Eq. (16) is employed, $\ln a$ and ln10 being located on the linear ordinate, $\log a$ and log10 on the logarithmic ordinate of semilogarithmic paper. An index line is drawn through the origin ($\log a = 0$, $\ln a = 0$) and the point ($\log 10 = 1$, $\ln 10 = 2.303$). This line intersects all points whose abscissas are $\log a$ and whose ordinates are $\ln a$. Hence, the natural logarithm of any number may be read by locating the number on the log scale and its natural logarithm at the intersection with the index line. The example shown in Fig. 20 is $\ln 17 = 2.83$. Proof of this construction is obtained by substituting in Eq. (3): $b = 0$, $y = \ln a$, $x = \log a$, and $m = \ln 10 = 2.303$; whence $\ln a = 2.303 \log a$, which is identical with Eq. (16′).

This relationship may also be employed to calculate logarithms to any base other than 10 or e. It is necessary to know only (1) the value of the modulus, $m_s = \log_s 10$ (where s is the base of the logarithmic system), or (2) the $\log_s$ of any number a, from which the modulus may be determined, since $m_s = \log_s a/\log_{10} a$.

Obviously, common logarithms may also be reckoned by this device, the index line being drawn from the origin (0, log1) through the point (1, log10). The linear scale will then give the values of logarithms of numbers on the log scale.

Logarithmic Paper

Logarithmic paper has rulings spaced logarithmically both horizontally and vertically. Its principal applications involve index lines which are best defined by the point-slope formula, Eq. (3). If the following substitutions are made—$b = \log B$, $y = \log Y$, and $x = \log X$, Eq. (3) becomes

$$\log B = \log Y - m \log X,$$

whence

$$B = YX^{-m}. \qquad (17)$$

18. Simple products. If in Eq. (17), $m = -1$, then $B = XY$, while if $m = +1$, $B = Y/X$. In

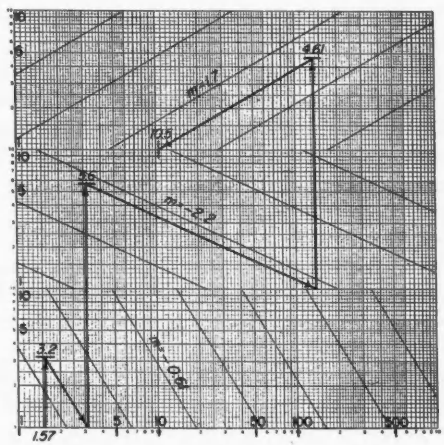

FIG. 24. Solution of the formula $x = 1.57a^{0.61}b^{2.2}/c^{1.7}$.

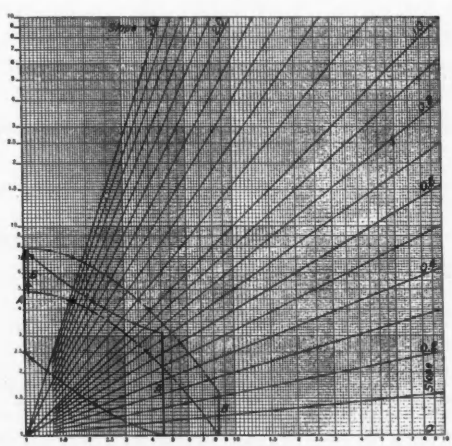

FIG. 25. A general method for formulas with fractional powers. Evaluation of $2.5\,(4.5)^{0.74}\,(8.2)^{-0.22} = 4.8$.

Fig. 21 at (*a*) is seen a sheet of logarithmic paper ruled with families of lines of slopes $m=-1$ and $m'=+1$, showing the evaluation of the product $23\times 8.2=188.6$ and the quotient $23/8.2=2.8$.

Extension of this principle to products of several terms is a simple matter achieved by taking the first product as the Y of a new product. This is illustrated in Fig. 21, at (*b*), which shows the operation $23\times 8.2\div 0.52=98.1$.

Division may be accomplished, as an alternate method, using the lines of slope, $m=-1$, by the device which amounts to finding that number X which will equal Y when multiplied by B. In Fig. 21, at (*a*), a reversal of the procedure followed for multiplication would provide the quotient $188.6\div 8.2=23$. This device, while providing a simplification (by eliminating one set of diagonals) for the method in general, has the principal advantage of offering a method for simultaneous multiplication and division. In evaluating the product, $B=XY/Z$, the pivot is moved along the diagonal ($m=-1$) from the intersection of X and Y to the ordinate Z, rather than the ordinate 1. The abscissa of the point is the value of B. This scheme is illustrated in Fig. 21, at (*c*), showing $42\times 21/29=30.4$.

A 45° triangle (or a T-square set at an angle of 45° to the axes of the paper) greatly facilitates the application of the above method and obviates the need of the diagonally ruled lines. Indeed, the use of these instruments provides a rapid, fairly accurate method of multiplication, which has the following advantages over the slide rule. (1) The method automatically fixes the position of the decimal point. (2) Commonly used constants are readily marked on the paper by horizontal lines, virtually eliminating the necessity of locating them during the calculation.

19. Products with fractional powers. If, in Eq. (17), m has values other than -1, then products with fractional or other powers result. For instance, by using slopes of -0.61, -2.2, and 1.7, the product $1.57\times 3.2^{0.61}\times 5.6^{2.2}/4.61^{1.7}=10.5$ is evaluated in Fig. 22. The primary problem in this method is not that of accumulating the product, but of setting the slopes. There are three convenient methods of accomplishing this: (1) Use of a "variable (or adjustable) triangle," on which the protractor has been calibrated with a scale of slopes (used in conjunction with the T-square, this device gives rapid readings easily); (2) use of a protractor or ruled card, scaled in slopes, in conjunction with a T-square or right triangle; (3) use of a chart of slopes to set the slope. The slope may be transferred to the graph paper by the T-square-triangle method of drawing parallel lines. Since these methods are obvious to those familiar with the use of drawing instruments, it is unnecessary to illustrate them here.

An elaboration of this method especially applicable to set formulas such as formulas with fixed exponents is shown in Fig. 23. The product obtained in the lower section is projected upon the Y axis of the section just above it, where it serves as a term in the new product. This product from the second section is then projected upon the Y axis of the third section, and so on until the product is complete.

To adapt this method to logarithmic polycycle paper, it is desirable to transpose X and Y axes, so that X axes are vertical and Y axes horizontal. This rearrangement must be taken into consideration in determining the slopes of the diagonal lines in each section. The method is illustrated in Fig. 23. In Fig. 24, application is made to the formula (see Sec. *16*):

$$x=1.57a^{0.61}b^{2.2}/c^{1.7}.$$

A third method of evaluating products with fractional powers is based on a transposed form of Eq. (17): $Y=BX^m$. If $B=1$, then $Y=X^m$. A family of lines of varying slopes is laid off on logarithmic paper. The value of any term, X^m, is then the ordinate of the line with slope m at a point whose abscissa is X. Accumulation of a product of such terms may be accomplished with dividers or an equivalent instrument. Each term, $X_1^{m_1}$, $X_2^{m_2}$, etc., in the product is "lifted" with the dividers and transposed to the Y axis, where it joins the products already there.

This method is illustrated in Fig. 25, showing the product $(2.5\times 4.5^{0.74})/8.4^{0.22}=4.8$. Obviously, this is another application of the method of evaluating a product by adding the logarithms of its terms.

20. Proportionality. Proportionality is established with logarithmic paper by constructing parallel index lines which intersect vertical or horizontal parallel scales. Proof of this method is identical with that given for semilogarithmic paper (Sec. *15*).

Twenty-Five Years of American Physics*

Karl K. Darrow
Bell Telephone Laboratories, New York, New York

WE have come together for a celebration of the twenty-fifth anniversary of the Physical Society of Pittsburgh, and therefore this Society must have been founded in 1923. I do not know all that was in the minds of the physicists of Pittsburgh in 1923, nor all of their incentives for founding a Society. However, I may suppose that they were such excellent prophets that they foresaw that physics was approaching one of the turning-points of its history, and they wanted to organize themselves to follow it around the turn. Or to use another metaphor: the physicists of Pittsburgh saw the buds which were ready to open on the tree of physics, and they banded themselves together to watch the flowering of those buds. By remarkable coincidence they had chosen a year which in physics was one of the closing years of an old dispensation, as were the years 1788 and 1913 and 1938 in world history. I must make haste to add that the analogy is not to be forced to extremes. The new things which began to happen in the world in 1789 and 1914 and 1939 were mostly evil, and the world sank into chaos. The new things which began to happen in physics in 1924 were mostly good, and as a result of them the science of physics has come closer to being an all-embracing theory than it ever was before—although what order of infinity yet separates us from that goal is something no man knows.

Let us try to project ourselves back into that quiescent year of 1923, and compare it as nearly as we can with these tormented times. It will be easiest to compare the *quantity* of production of physics. Quantity has this advantage over quality, that it can always be measured; for that in effect is just what the word "quantity" means. I will take as a measure of quantity the number of pages published per year in the various journals of physics published in the United States: and since the year of 1948 is not yet over, I will compare 1947 with 1922. In so doing I shall be discriminating in favor of the past over the present, for in 1947 we were not yet so far recovered from the latest war as we had been in 1922.

Here is our central journal of physics the *Physical Review*, with 2248 pages in 1947. Here is its companion the *Reviews of Modern Physics*, with 364 pages (in 1948 it will be considerably thicker, owing to the special issue honoring R. A. Millikan).[1] Here is the *American Journal of Physics*, organ of the American Association of Physics Teachers, with 524. Here is the *Review of Scientific Instruments*, with 954. Here is the *Journal of Applied Physics*, with 1152. Here is the *Journal of the Optical Society of America*, with 1012. Here is the *Journal of Chemical Physics* with 900 pages, and the *Journal of the Acoustical Society* with 1036. Total, eight thousand one hundred and ninety pages.

Now there by contrast are the journals of 1922. There is the *Physical Review* with 1401 pages, and the journal bearing the double name of *Journal of the Optical Society and Review of Scientific Instruments*, with 1083 pages. Total, two thousand four hundred and eighty-four, or about thirty percent of the sum for 1947. And there is—but that is all there is, there isn't any more. None of the other journals of 1947 existed in 1922. Somebody may cavil that in 1922 there were other journals not entirely devoted to physics, in which, however, some papers on physics appeared. So there were then, and so there are now. I think that if I had gone through or delegated the considerable labor of taking due account of all, the ratio 3 to 1 would not have been significantly changed. Yet it was already in 1923 that one of the most distinguished of American physicists put an end to his life, leaving a very poignant note in which he wrote "Physics has got away from me, and I cannot catch up." If everyone felt as acutely as he our common limitation, the problem would have disappeared along with the people who confront it.

Another measure of the growth of physics is

* Address delivered on the occasion of the twenty-fifth anniversary of the Physical Society of Pittsburgh, October 7, 1948.

[1] I seize the chance afforded by belated publication of this speech to remark that in 1948 *Physical Review* jumped up to 3470 pages and *Reviews of Modern Physics* to 728.

the size of the American Physical Society and of its meetings. Just before Pearl Harbor, seven years ago that is to say, the American Physical Society had somewhat fewer than four thousand members. When at the Chicago meeting of next month, November 1948, the Council gets through with electing the newest candidates to membership, there will be somewhat over seven thousand eight hundred. Before the war our biggest meetings had fewer than nine hundred registrants, and there was only one a year on that scale. Since the war our biggest meetings have had over seventeen hundred registrants, and there are two a year on that scale. If there is anybody who prefers a meeting of these dimensions to those of the olden time, I have yet to make his acquaintance; and yet the physicists come.

Now I will try to impersonate a physicist of 1923 standing up behind a lecture-table and trying to impart a general view of his science to an audience such as you. Most of those in this room cannot remember such a phenomenon, but I can. I was nearly such a phenomenon myself, for I gave a course of three lectures in this city of Pittsburgh only two years after 1923. My lecture-notes of 1925 have vanished, but I am sure that I can say the same things without even thinking about them.

I undoubtedly began in 1925 by speaking in much the same way as though I were addressing an audience of laymen this very evening. I had a blackboard and on that blackboard I must have made a large blob to represent the nucleus of a hydrogen atom and a small dot to represent a negative electron. The nucleus is a positively-charged particle which has nearly all of the mass of the atom—all but one part out of 1850, in the case of hydrogen. The electron is a negatively charged particle which is running around the nucleus in an elliptical orbit according to the laws of Kepler. It cannot however, run around in just any dynamically-possible orbit: it is obliged to choose among particular discrete orbits. These are fixed by peculiar stipulations known as quantum-conditions. To each of these orbits corresponds a certain value of the energy of the atom which we call a "permitted energy-value."

You perceive that I am now in the groove (a locution which did not yet exist in 1923) and I could probably go on like this for an hour or so while thinking about something else. But the only point I want to make is, that in this year of 1948 we should probably all talk in about this way to laymen, but we should experience a certain reluctance to talk this way in the presence of a bright young graduate student of theoretical physics. We have in fact a situation similar to that of various religious orders of long ago, which had an *esoteric* language for the initiates and an *exoteric* language for the common crowd outside the gates.

If a physicist were to make the mistake of speaking in the esoteric language to an audience of laymen, he would begin by marking on the blackboard the large blob for the nucleus and the little dot for the electron. So far as I know, even the initiates have never got rid of these. But then he would write down a differential equation of the second order, in which the energy of the atom would appear as a constant. He would say that the permitted energy-values of the atom are the eigenvalues of this equation; and he would explain what is meant by "eigenvalue," using such words as "continuous" and "single-valued" and "finite." Next he would write some eigenfunctions on the blackboard. By this time the audience would be hoping that the speaker had some lantern-slides so that it could slip out under cover of darkness. The collective disquiet of the audience would eventually react upon the speaker, assuming—as I should hope would be the case, but I should not take it for granted—that the speaker had a non-zero sensitivity to the mood of the audience: and either he would switch suddenly over into the exoteric language, or the lecture would finish in a slough of despond.

But who are these hypothetical laymen, who are so much interested in physics that they want to hear lectures about it, and yet cannot absorb what is said in the esoteric language? Well, the strange and funny point is that by and large, these laymen are ourselves. We know that the proper way to find the permitted energy-values of the atom is to use the differential equation; but having used it or at least having found out how to use it, we talk about the orbits of the electron. We even visualize the orbits of the electron, so as to have something to hold onto as well as something intelligible to talk about. We use one technique for getting the results and another for

keeping them in mind. Now this sort of thing came into being only in 1925 and the years thereafter. When the Physical Society of Pittsburgh was established, there was still only one language. This Society was founded just in time to witness the beginning of double-talk among the physicists. Or in more imposing language, 1924 marked the end of the classical quantum mechanics.

I must qualify my statement somewhat by admitting that we did have a small amount of double-talk even before 1923, in the days when we were becoming aware that light has both the properties of corpuscles and the properties of waves. At that time, however, the ambiguity was confined to light and did not extend to electricity and matter. Now here is a place where I can properly quote the achievements of American physicists. If you want to illustrate the doctrine that light possesses some of the qualities of corpuscles, you can find no more sharp and cleancut demonstration than that which Arthur H. Compton afforded when he interpreted the Compton effect—the scattering of x-rays by electrons, with change in the quality of the x-rays. I have to admit that this discovery was made just at the end of 1922, but perhaps I may be permitted to force it into the scope of this lecture by recalling that as lately as 1923 there still were doubters. If you want to illustrate the doctrine that electrons are invested with some of the qualities of waves, you can find no more sharp and cleancut demonstration than that which C. J. Davisson afforded when he diffracted electrons by crystals of metal. (Let it be recorded here that both Davisson and Compton had worked in Pittsburgh.) It is a singular coincidence that two such contrasting and yet allied discoveries—two *complementary* discoveries, I suppose that Bohr would say—should have been made in the United States. Thus for both light and electrons, we have to use the language of corpuscles and also the language of waves. What we need is a single language supplanting both; but of this even the grammar has yet to be written.

Using the language of particles, I will now take up the status of the particles called "elementary." Of course, you will not expect me to speak exclusively of American discoveries, regardless of the title of this talk. If some complicated crossword puzzle had been solved by several people working jointly, you would not expect a historian to make much sense out of the story by mentioning only the words which were guessed by one only of the several.

When the Physical Society of Pittsburgh was founded, it was believed that nuclei consist of protons and negative electrons. Add on the negative electrons which circle around the nuclei, and still you have only two species of particles. Add on the photons or corpuscles of light, and you have three. The world into which this Society was born consisted accordingly, in the minds of the physicists, of particles of three kinds only. This is a nice small number. I wish we had been able to keep our threefold world, but we were not.

The first of the intruders into the threefold world was the *neutron*. It would be an interesting task for a graduate student of the history of physics—supposing there to be such a person—to search out all the passages in the literature of physics where the word "neutron" was used in the 1920s and even earlier, and then judge whether any of these passages contained a veritable prophecy. Not having done this myself and not knowing of anyone who has done it, I abide by the general belief that the neutron came into physics, not with any American aid, at the very beginning of 1932. It appeared first as a free particle accessible to observation, and then in a few months it entered physical theory as a constituent of the nucleus. At present we consider nuclei as consisting of protons and neutrons. This means that the neutron has evicted the negative electron from our model of the nucleus. If the universe consisted entirely of nuclei and light, we should still have a threefold world with photons, protons and neutrons as the members of the trinity. But the world does not consist entirely of nuclei and light, and it still comprises negative electrons. Therefore at the beginning of 1932 we had briefly a fourfold world. Nevertheless the neutron has on the whole been a welcome interloper, and I am sure we should not like to do without it.

The next intruder was the *positive electron.* This was brought into the picture of the world in August 1932 by an American physicist, C. D. Anderson, who found the first example of this particle wandering in the air, a product of the cosmic rays. This was the first instance in history

of someone discovering an elementary particle by observing just one of it, instead of some aggregate effect of a large number of such particles. The positive electron raised to five the number of elementary particles, and endowed us with a fivefold world. Yet he again was a welcome interloper. He brought about a sort of symmetry in the world-picture. People had wondered vaguely why all electrons should be negative, all protons positive. Here came a particle which showed that electrons could be either negative or positive. No such symmetry has been established for the proton; but the positive electron represented a step in a welcome direction. The same cannot be said of the next kind or kinds of particles with which the insatiable Anderson supplied us.

These are the particles which have borne so many names, that even yet no great majority of physicists agrees on any. The four favorites are meson and mesotron pronounced with a short *e*, and meson and mesotron pronounced with a long *e*. A small though influential group is promoting mesoton, but my guess is that this name is going to follow yukon and barytron and heavy electron into oblivion. I do not know why nobody has yet suggested that the name be ***mess-on***, with a double *s* and the termination taken to mean that the situation is a continuing mess.

You notice that I have referred to the mesons in the plural number. This is because there are at least two values of mass represented among them, and very possibly more than two—a fact not apparent for fifteen years after Anderson's discovery. Since for each value of mass there are both positive mesons and negative, we have now at least four kinds of mesons, making at least a ninefold world. These bodies have none of the pleasant attributes which made welcome interlopers of the positive electron and the neutron. They have not enhanced either the symmetry of the universe or the simplicity of our nuclear models. I read somewhere once that some of the early astronomers objected to comets, not on grounds of superstition but because the comets spoiled the integrity of the solar system. On this basis the mesons are the comets of modern physics, for they have spoiled the integrity of the atomic system. And even they are not the worst.

In addition to all of the foregoing particles we have now the ***neutrinos***. The reason for these I may crudely illustrate as follows. Suppose that you were to project a billiard ball across a table, and right in the clear and open middle of the table you were to see it undergo a sudden sharp deflection and a loss of speed. After excluding the possibilities of a hole in the table and of an earthquake, you might conceivably come to suppose that there is a category of perfectly invisible billiard balls, with one of which your visible ball had collided. This might seem a fantastic theory. Still if you were to hear the invisible billiard ball thumping against the edge of the table, or see another visible ball on the table suddenly leaping back from its impact, you might feel very confident of it. Unfortunately this clinching sort of evidence is just what is still wanting in the case of the neutrino: and so long as it continues to be wanting, this ghostly particle will continue to seem unpleasantly ghostly—all of which does not prevent the physicists from believing in neutrinos. Assuming that there is only one kind of these, which some people regard as too narrow an assumption, we have now at least a tenfold universe.

More disagreeable yet is the fact that these particles called "elementary" have foregone the attribute which used to be considered the most important of all. In the older atomic theories, it was the prime business of an atom to be eternal. Change was confined to the relations of the atoms with each other. Our newer elementary particles seem to be in a frantic hurry to get themselves changed into others: positive and negative electrons into photons, photons back into pairs of electrons, mesons into electrons or into other kinds of mesons and neutrinos. As the ***Book of Common Prayer*** says about man, the new-fangled elementary particle never continueth in one stay.

Constancy has not however, entirely vanished from physics: witness the tables of the "general physical constants" which R. T. Birge has published from time to time since 1929, containing not only the values of these which he judges most likely to be right, but also the uncertainty with which each of them in his judgment is afflicted. It is not often that the world accepts as final the verdicts of an American judge, particularly when the judge is also the jury; but in this case the practice antedated the United Nations.

Most of the facts which I have been citing were

derived from the study of stable isotopes, from transmutation and from research in cosmic rays. Let us see how American physics has been faring in these fields.

The glory of the discovery of isotopes belongs to England in the persons of F. Soddy, J. J. Thomson and F. W. Aston. Aston, with the opportunity of the forerunner, discovered more stable isotopes than any other man, with Arthur J. Dempster of Canada and the United States in the second place. The most celebrated of the stable isotopes belongs however to Harold C. Urey, the discoverer of heavy hydrogen better named deuterium. Our physicists have designed and set up more than their share of the various mass-spectrographs delicately devised for measuring the masses of atoms very precisely; and we have pioneered, for a necessary though not a pleasant reason, in the art of separating isotopes on the grand scale.

Since I have just spoken of the discovery of isotopes, this seems to be the place for inserting our claims to the discovery of elements. We claim 96 curium, 95 americium, 94 plutonium, 93 neptunium, 85 astatine, and 61 prometheum. The only counterclaims are due to people who have examined the x-ray emission spectra of various minerals, and have believed—as they may still believe—that they identified various lines as belonging unmistakably to one or another of those elements. This is not a question for the inexpert to settle. I can only report that the reverberations of the counterclaims appear to be dying away as the years go on. If they are destined to die off into silence, the United States can boast of six among the eight new elements discovered since the Physical Society of Pittsburgh was founded: of the two others, 87 francium belongs to the nation which its name recalls, 43 technetium is due to Italy with the aid of America.[2] All six, and technetium also, were created—that is to say, radioactive isotopes of these seven were created—in the laboratory by the art of transmutation, and all were first detected by their radioactivity though some among them have by now been made in abundance great enough to be seen and weighed.

Transmutation likewise was born in England—born twice, it is fair to say: once when Rutherford effected it with natural alpha-particles, again when Cockcroft effected it with protons accelerated by a high-voltage machine (how odd it now seems to refer to half-a-million volts as "high voltage"!). The first event occurred five years before this Society was founded, the second six years after. It was not until 1932 that Ernest O. Lawrence reft the primacy from England by putting to work the first of the cyclotrons, which has had so many descendants that the United States is now as distinctively the land of the cyclotron as it is the land of the skyscraper. Perhaps now it is even more distinctively the land of the cyclotron, since the cyclotron-builders are still designing and building grander and grander examples, a stage which the skyscraper-builders seem to have left behind them eighteen years ago. It is also the land of the betatron (D. W. Kerst), the synchrotron (E. M. McMillan), the linear accelerator (L. W. Alvarez) and the electrostatic generator of R. J. Van de Graaff which some people want to call the statitron.[3] I should like to suggest that the last and most monstrous of these machines be called the omegatron: but it is reserved for future generations to know which instrument this will be.

Owing to these magnificent instruments and to the funds and people available to work them, and owing also to the pile (first constructed in America but not entirely by American-born or American-trained physicists), we have now to our credit a much larger share of the reactions of transmutation, of the artificial radioactive isotopes, and of the measurements of the things unromantically known as "cross sections," than any other nation. Yet here I must impersonate the slave who used to ride behind the general in the Roman triumphs, muttering into his ear at suitable intervals "Remember that thou too art mortal," or words to that affect. What I have to mutter at this point is that artificial radioactivity was discovered in Paris, transmutation by slow neutrons in Rome and the proper interpretation

[2] It was discovered at Palermo in molybdenum irradiated at Berkeley with deuterons. I have not given the names of the discoverers of these elements, as each element (excepting 87) is associated with two to four names. See G. T. Seaborg, *American Scientist*, **36**, 361 (1948).

[3] The names here associated with the betatron, the synchrotron, and the linear accelerator are those of American physicists who have greatly developed them.

of fission in Berlin and Copenhagen. Truly the battle is not always to the apparently strong.

As to the cosmic rays: to begin with, America in the person of R. A. Millikan supplied the name. This may sound like a disparagement, and we have been assured by Shakespeare that a rose by any other name would smell as sweet; but I remember the time when people spoke of "penetrating radiation" instead of "cosmic rays," and I suspect that if that name were still in use the popular interest in the subject would be very faint and even that research would have been slowed down. This Society was founded just when everyone had come to agree that a part of the ionization of the atmosphere is ultimately due to something coming from beyond the atmosphere. Please note the careful wording of that phrase. In 1923 it had only just been settled to everybody's satisfaction that *part of the ionization of the atmosphere is ultimately due to something coming from beyond the atmosphere.* Thirty years of study of the penetrating radiation had brought us just so far!

You see that the phrase implies a distinction between the immediate ionizing agent which detaches the electrons from the molecules of the air, and the ultimate cause or "primary rays" which produces the immediate ionizing agent. The most obvious thing to imagine is that the immediate ionizing agents are negative electrons and the ultimate cause is of the nature of x-rays or gamma-rays or super-gamma-rays—in other words, photons of high frequency light.

Many of us remember the controversy which enlivened the decade of the twenties, contributing in no small measure to the progress of cosmic-ray research. One of the schools defended the position which I have just described. The other school was ready to admit that some at least of the immediate ionizing agents could be negative electrons, but it contended that the primary rays were charged particles coming out of interstellar space, and it did not exclude the possibility that some of these primary particles might barge down into the lower strata of the atmosphere and do some ionizing on their own.

Now we know that, in the vulgar language, neither school had seen anything yet. Seeing of cosmic-ray particles began in the decade of the thirties, when first the cloud-chamber and then the photographic plate were put to work. Then and there began the era of the magnificent photographs of cosmic-ray showers, cosmic-ray stars and isolated tracks, which converted the study of cosmic rays from a laborious study of curves to the viewing of a wonderful museum of fascinating pictures. The trails of the immediate ionizing agents are there for all to see. It would be very unwise for anyone but an initiate to try to identify them, but the initiates tell us that they can recognize the particle from its track, and that among the particles are electrons of both signs, mesons of both signs and of at least two masses, protons, alpha-particles and still more massive nuclei. Nearly all of them are born in the atmosphere itself, mostly out of blocks of lead or other metals which are exposed close to the cloud-chamber or in the chamber itself. This brings out a defect in the title "cosmic rays," of which a speaker is likely to become acutely conscious when he has to tell an audience of laymen that nearly all or quite all of the particles of which he shows the trails have never been anywhere outside the atmosphere. At this point of the argument the primary or *truly* cosmic rays tend to recede into the status of a nebulous hypothesis. However, thanks to the balloons and the high-flying airplanes which have lately carried the apparatus of the physicists (with or without the physicists themselves) into the higher reaches of the atmosphere, it now seems very probable that we have some photographs showing the tracks of the primary particles themselves.

Luckily we do not have to depend upon these tracks to prove the nature of the primary radiation. The intensity of the cosmic rays depends on the geomagnetic latitude of the place on the earth's surface where you observe them; and the number of immediate ionizing particles coming from east of the zenith is different from the number coming from west of the zenith. This "latitude-effect" and this "azimuth-effect," as they are called, could not occur if the primary rays were uncharged, for they are due to the effect of the earth's magnetic field upon the moving charges. Here I would like to mention the contribution made by Mexico, in the person of M. S. Vallarta, to the observation and analysis of these effects, the latter of which demands no mean

mathematical technique. Also in farewell allusion to the once-famous controversy now extinct, I mention that *both* of the one-time antagonists Millikan and Compton journeyed far and wide to measure the latitude-effect; Millikan at one time thought that there was no such thing, but it turned out that by a singular fate his first voyage had been across a part of the earth where the intensity of the cosmic rays is strangely and deceptively nearly unaffected by latitude.

So far as American contributions are concerned, I believe that by far the most of the cloud-chamber photographs of cosmic-ray phenomena have been made in the United States, and also by far the largest number of observations at great altitudes. Austria was, however, the first to exhibit the tracks of cosmic-ray particles embalmed in the emulsions of photographic plates; and England was the first to produce photographic emulsions capable of showing the tracks of mesons, to which invention we owe most of the basis for our present conviction that there are at least two kinds of mesons, of which the heavier converts itself into the lighter. Here is a nice illustration of the international character of science: the Brazilian physicist Lattes learned at Bristol in England the art of developing these emulsions and brought it with him to Berkeley, whereupon it was found that the new 184-inch cyclotron was busily producing mesons with its 380-million-volt alpha-particles. This at least is definitely an American priority. If we still used the Latin language we would confer upon the Berkeley physicists the title of *mesonifices*.

Since we have been up in the air for the last few minutes, let us not come down without ascending into the ionosphere. Here we find something which is never classified with the cosmic rays, and yet it answers perfectly to the definition of cosmic rays though not to that of penetrating radiation. This is the vast multitude of slow electrons which are detached from atoms by various agents coming out of space, first and foremost the ultraviolet light of the sun; and which remain detached long enough to absorb and to reradiate the radio waves which we project against them from our sending stations. The best method for studying these is the radar method, in which pulses of radiofrequency waves are directed into the sky and we observe their echoes. The name "radar" is a product of World War II, and yet this method was invented by Gregory Breit and M. A. Tuve in 1925, the first year of the existence of this Society. This is a vast and flourishing field of research, and yet if you were to confine your reading to the *Physical Review* you would hardly suspect that it even existed. This is because the literature of the ionosphere has fled from the journals of physics and has found a haven in the *Proceedings of the Institute of Radio Engineers* and other similar journals. Considering how much we physicists have to read as matters stand and how much we have to pay for publishing what we read, it seems judicious not to try to recapture the ionosphere for our journals.

Now it is positively time to delve into the solid state, a region where the theoretical physicist has never been quite so happy as when he could deal with an atom wandering around all by itself. At the moment he is probably least unhappy among the metals. When this Society was founded there was a theory of the conduction of electricity and heat in metals: it was the theory of Drude and Lorentz, and we now call it "classical." This was the theory that the interstices between the atoms of a metal are pervaded by a gas of free electrons. The career of one of these electrons is an alternation between free flights and sudden bumps. In the absence of an electric field the electrons fly equally in all directions with the high speed of thermal agitation. In the presence of an electric field they still retain the speed of thermal agitation, but on it is superposed an extra speed which the field imparts to them. This theory explains Ohm's Law, but fails to account either for specific heat or for the dependence of resistance on temperature. Everyone was acutely aware of these defects in 1923, but nearly everyone felt that the theory just had to be right, and we went on teaching it.

Well, the theory *was* all right, but it had to be accommodated to wave mechanics. In a German journal of 1927 there is a series of papers by Arnold Sommerfeld of Munich and his students, in which the accommodation is made. Sommerfeld himself abolished the difficulty with the specific heat, by applying to the electrons what is known as the Fermi-Dirac statistics; but it remained for one of his students to settle the difficulty with the resistance, and this student was an American—

W. V. Houston. The difficulty vanishes when it is realized that electrons are guided by waves. This, I pause to remark, is an example of the defective grammar of the temporary language or pidgin-physics with which we are obliged to fumble, in our attempts to describe things which have both the properties of waves and the properties of particles. Bear with me while I use it; and remember that waves are not scattered or in any way affected by perfectly regular lattices provided that the wavelength of the waves be greater than the spacing of the lattices as is the situation here. But since the guiding waves are not affected, the electrons are not affected either. A perfectly regular lattice of atoms would therefore be transparent to the conduction-electrons: a substance with such a lattice would be without resistance. But the atoms wiggle about their mean positions because of thermal agitation; and this eliminates regularity and introduces disorder, and with disorder comes resistance. Houston showed that the theory gives the right dependence of resistance on the temperature.

Resistance therefore is due to disorder: this is one of the two new principles (the other being the Fermi-Dirac statistics, itself reposing on the exclusion-principle of Pauli) which have transformed the classical theory of electricity in metals. If anyone wants to exhibit this principle vividly, the best way of doing it is to speak of equimolecular binary alloys. There are some of these in which the metallurgist can create at will either an orderly alternation of the two kinds of atoms or a complete disorder—not, I repeat, a disorder resulting from thermal agitation, but one resulting from the fact that there are two kinds of atoms randomly strown over the lattice-positions. The state of disorder has a markedly greater resistance than the state of order.

There is something else in physics which is correlated with disorder, and its name is *entropy*. The new statistical mechanics provides us with a formula for the entropy of a gas which can be tested by experiment, contingent on the assumption that the entropy of the solidified gas is zero at the absolute zero where disorder is nil. American origin cannot be claimed for the formula, but most of the experiments by which it has been verified have been made in the United States, and most of these by Giauque and his school at Berkeley.

A magnetizable substance enjoys (if that is the appropriate verb) a type of disorder peculiar to itself: the disorder in the orientations of its elementary magnets relative to an applied magnetic field. This peculiar disorder may be reduced by applying a very strong magnetic field, which brings the substance near to magnetic saturation. Now after equilibrium is reached, let the field be suddenly canceled. The elementary magnets return to their random orientations, and the disorder increases and so does the entropy. We are taught that increase of entropy usually goes with inflow of heat, but in this case the inflow of heat cannot be quick enough, and the magnetizable substance cools down—down to temperatures lower than can be attained in any other way. This is the method of *adiabatic demagnetization*. Giauque in California and Debye in Switzerland thought of it at about the same time, but it was Giauque who first made use of it; and for a while we had in this country the lowest temperatures of record.

Some of you must be wondering why I have not yet spoken of the tide which flooded the *Physical Review* in the decade of the twenties, and ebbed thereafter to a trickle. This was the flood of papers on optical spectroscopy—spectroscopy, that is to say, of light, omitting the radiations from nuclei and the radiations from antennae. The theory of optical spectra is pretty nearly identical with the theory of atomic and molecular structure. In its heyday the spectra of all the elements were analyzed and classified according to their multiplet structure; the investigators found the terms, of which the frequencies of the lines were the first differences; the terms were labeled with symbols to indicate the orbits in which the electrons were supposed to be revolving; and there was scarcely a term too poor to find someone to do it the reverence of a paper or a Letter to the Editor of the Physical Review.

Pretty suddenly the tide rushed out (to the discomfort of, among others, the astrophysicists), and nuclear physics came in. If you read the history of California you will know of a man named Sutter, who just a century ago was farming an immense estate near Sacramento with the help of pioneer farmers. Came the discovery

of gold in the Sierra Nevada, and all the farmers deserted the estate and rushed off to the gold-fields. In the language of this parable, spectroscopy was Sutter's estate and nuclear physics is the gold of the high Sierras. Yet I have the impression that the parable is not fair, because the estate of spectroscopy was harvested and there was little left but gleanings when the gold-rush started. In other words, the theory of atomic structure followed the path of celestial mechanics: so thoroughly and well had its major problems been solved, that there was nothing remaining over except a host of minor problems of which the difficulty was in inverse ratio to the importance. Band-spectroscopy and the theory of molecular structure constituted a sub-tide (if I may coin the word) which did not rise so high and has not ebbed so far. I conjecture that this is because the theory of molecular structure is so much harder than the theory of atomic structure that some of the outstanding problems are important enough to justify the labor. In this field America possesses an outstanding authority in the person of R. S. Mulliken.

You will remember the part in spectroscopy which is played by the Zeeman effect, and you will remember how this is interpreted in quantum theory. In the exoteric language, if an elementary particle has a magnetic moment and is bathed in a magnetic field, it is allowed to point only in one or another of a very limited number of "permitted" or "quantized" directions. It is very easy to calculate just how much work you must do upon the elementary particle to swing it from one of these permitted directions into another more nearly opposed to the field. You can verify your calculation by analysis of spectra, but this is not the most direct conceivable way to do it. The most direct conceivable way consists in exposing the elementary particle to a strong magnetic field, and simultaneously exposing it to light of which the photons have exactly the energy corresponding to that amount of work. Under such conditions you might expect to find a strong and sharp absorption. But when you figure it out you discover that the light which is required is in the radiofrequency range. This was an obstacle not overcome till barely three years ago.

The phenomenon of which I am now speaking is called *magnetic resonance*. The magnetic resonance of electrons in paramagnetic solids was first observed in Russia, that of electrons in ferromagnetic solids was first observed in England; both have since been studied more extensively, and in the ferromagnetic case much more understandingly, in the United States than elsewhere. There is also *nuclear magnetic resonance*, discernible in solids and in liquids and in gases. This is a contribution of the United States, having been discovered simultaneously by E. M. Purcell and Felix Bloch at opposite ends of the country. It teaches us about the magnetic moments of nuclei; it teaches us about the rates at which nuclei of various kinds come into thermal equilibrium with the magnetic field and with the substance which environs them; it even teaches us facts about crystal structure, and probably it has a good deal more to tell. It is worth while to ponder on the fact that this information about *nuclear* physics was discovered through the perfecting of *radiofrequency* techniques, part of these developed at the great wartime Radiation Laboratory of the Massachusetts Institute of Technology.

It may have been noted that I took care to speak of the recent discovery of magnetic resonance in *solids and liquids and gases*. This was because magnetic resonance was first observed eleven years ago, in the course of the long and continuing sequence of experiments on molecular beams contrived and done by I. I. Rabi and his numerous students, many of them now working on their own. This is work unparalleled elsewhere. It is marvelous to observe how instructively a beam of neutral molecules or atoms, very narrow and perhaps a metre long, may be made to disport itself in a nonhomogeneous magnetic field to which a radiofrequency field may or may not be superadded. These experiments have taught us the magnetic moments, the spins and the quadrupole moments of multitudes of nuclei, and their couplings with the electrons which revolve around them. Very lately they have shown that the magnetic moment of the electron and the energy-values of the stationary states of hydrogen are not quite what the extant theory predicted, and this has led to striking new developments in what is called *quantum electrodynamics*. This imposing term overawes me as

much as it does you, and we will not continue further along this line. Let me only reiterate that here is an outstanding example of a whole field of research developed and inspired by one man. There are many more, but the limitations of knowledge and of time debar me from mentioning more than the forty-year series of researches on the properties of matter performed by P. W. Bridgman, the work of D. L. Webster and Paul Kirkpatrick on the absolute intensities of x-rays from electron-bombarded metals, the work of L. B. Loeb on the spark and the work of Irving Langmuir and his colleagues on the arc.

Now this is really about all that there is time to say; and I must offer my apologies to the multitude of notable American physicists whom I have not even mentioned. At this point I feel, rather like Newton, who said something to the effect that he felt like a child who had been picking up pebbles on the beach while the ocean of truth lay unexplored around him. I can only pray that the eminent physicists whose diamonds I have not picked up will not avenge themselves by throwing stones at me. But after all I have praised a good deal of American physics, and here comes again the Roman slave; what is he muttering now?

The Europeans have long been wont to say that Americans excel in works of mass-production and mighty engineering, the implication being—and it is not always a veiled implication—that in delicate and subtle matters we are weak. Probably everyone here would like to decry that idea. But how embarrassing it is that when we think of American physics there come to mind the giant cyclotrons and synchrotrons and piles, the high voltages, the high pressures, the consummate equipment, and the huge amount of detail-work possible only in a country rich in population for providing students and rich in funds for paying for their work! It would be gratifying to redress the balance by quoting a number of great phenomena and principles of physics which were discovered in our land.

You know how a night-club or resort hotel allures its customers by featuring a "name band." Let us review the roster of "name principles" of physics, omitting all which are more than fifty years old so as not to weight the balance too much against the United States. Here are the Lorentz electron-theory—the Planck quantum-constant—the Einstein relativity—the Rutherford atom-model—Laue patterns—Bragg diffraction-beams—Bohr's correspondence-principle—the Bose statistics—the Fermi-Dirac statistics—the de Broglie waves—the Pauli exclusion-principle—the Schroedinger wave-equation—the Heisenberg uncertainty-principle—the Raman effect—the Weiss domain-theory—the Brillouin zones. Ransacking my memory, I have found no counterpoises to all these beyond the Compton scattering and the Russell-Saunders coupling, the Franck-Condon principle of molecular spectroscopy and the Gurney-Condon theory of alpha-particle emission, the Patterson diagram and the Oppenheimer-Phillips process. I could have done better if Americans had made a practice of adopting certain names which would be amply justified, such as the Davisson effect, Rabi resonance and Houston's principle. Even so, the contrast still would be embarrassing to those who esteem American physics. I hope it will be mitigated when my successor comes in 1973 to speak before the crowd which will assemble for the fiftieth anniversary of the Physical Society of Pittsburgh.

Let us imagine a thinker whose powers of penetration enable him to pierce through the obscurities of Eddington's diction to the underlying truth; to realize also, with equal clearness, the more orthodox points of view of Einstein, Bohr, Dirac and others; to see wherein the outlooks differ and how they can be reconciled; and to re-express the work of all in terms compellingly obvious, so that the contribution of Eddington can be assessed at its true value, and physicists everywhere united on a common front in an immensely strengthened position for apprehending and solving the problems that remain. There can be no doubt, I think, that such a man would render to physical science, the greatest benefit possible to it at the present time, yet the world of physical science has no provision for acknowledging such a benefit. For slightly inferior intellects there are Copley Medals, Orders of Merit, Nobel prizes and the like, but for him, nothing. Like Bunyan's Interpreter, he makes pilgrimage possible, and is heard of no more. . . . If he cared to fulfill certain other conditions well within his scope, he could earn an M.Sc. degree of London University in the History and Philosophy of Science.
HERBERT DINGLE. "The Missing Factor in Science." H. K. Lewis & Co. Ltd., London (1947).

A Report on the State of Physics in Germany

M. von Laue
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THE President of the American Physical Society has asked me to report on the state of physics in Germany. I extend my sincere thanks for the invitation, but such a report is confronted by two great difficulties: first of all, everything in present-day Germany is dependent on politics to an extent which can hardly be imagined; and, after all, I do not want to enter on a detailed discourse on the political situation. The enormous confusion characteristic of the German situation today is reflected also in the scientific sphere. Secondly, even for someone living in Germany it is not easy to form an over-all picture of the academic situation. The press pays scant attention to the subject, and such reports as may appear have little reliability. In some localities one has to exert the utmost caution in the choice of words in personal correspondence; postal communications everywhere are still slow, and in the case of two towns situated on opposite sides of certain zonal frontiers, are sometimes at a standstill for many weeks at a stretch. Therefore, one has to depend for scientific news mostly on oral reports of colleagues who happen to pass through one's locality on their travels, and on what one can see and hear on a journey. Traveling is just what everyone tries to avoid, because, at least up to the time of the currency reform, trains were very overcrowded, and a legal crossing of the zonal frontiers required an inter-zonal pass which was hard to obtain. Fortunately, the latter obstacle has for some time now ceased to be troublesome for the American-British zonal border, and recently, since August 1, 1948, for the frontier of the French Zone. At any rate, you will understand that my report cannot lay claim to comprehensiveness; I ask you to regard it as a mirror reflecting the general atmosphere. I ought to mention the fact that I left Germany two weeks after the introduction of the currency reform, and that since that time letters have been my sole source of information on the miscellaneous events in academic life. Thus, it may be that part of what I am saying to you no longer applies to the present situation.

Physics in Germany since 1933 has suffered heavy losses in personnel. How many scientists have emigrated you know very well, since the United States and England have given refuge to the majority of them. Only a few physicists fell in action—among the better-known only H. Euler, who worked with Heisenberg on cosmic rays. Some died during and after the war, among them Hans Geiger who succumbed to a prolonged, serious illness. W. Kolhoerster met with a traffic accident, and W. Nernst, F. Paschen, P. Lenard, and Max Planck died at the summit of their years. In May, 1945, American officers took Planck and his wife by car from the battle area at the Elbe to Göttingen, where they found refuge with relatives. Their total possessions comprised the contents of a small suitcase and a rucksack. They suffered deep mental anguish over the loss of their son who was executed in January 1945 as a participant in the July 20 conspiracy. Right up to his death on October 4, 1947, Planck still undertook numerous lecture tours, for teaching was the very breath of life to him. At his solemn interment at the Municipal Cemetery of Göttingen the whole city was present.

Among those better-known physicists who went to Russia, in addition to numerous members of the Physical-Technical Reich Institute, the Siemens Works, the Zeiss Works, and the firm of Telefunken, are G. Hertz, M. Volmer, H. Pose, K. Doepel, H. Scheffers, and H. Thiessen. G. Hertz and a number of others reside South of the Caucasus at the Black Sea. All of them can maintain only scant contact with relatives and friends in Germany with the aid of intermediaries.

There are still in Germany today several physicists who have not yet been denazified and therefore cannot accept academic office, but of whom it can be assumed that they will soon return to academic life. Unfortunately, there are also instances involving seriously incriminated persons,

in which the denazification procedure has miscarried. Therefore, the German Physical Society in the British Zone asked its Council in September, 1947, to take measures preventing the return to academic life of persons denazified in default of justice. The Council acted in accordance with this request in several instances.

I can think of no better way of acquainting you with the present whereabouts of important physicists in Germany than by listing the names of those with whom you are likely to be familiar alongside the institutions to which they are now attached. The list is broken into four sections, one for each of the four occupation zones of Germany. You miss in my list the formerly famous University of Leipzig; the reason is that its Science Faculty has disintegrated. A question mark indicates that the status of Giessen is in doubt; I do not know if it is still a university or if it is now established as some other kind of school. Brackets enclosing a scientist's name indicate that he has retired from active work.

I. American Zone and American Sector of Berlin

a. **Universities**
- **Erlangen:** Hilsch, Molwo, Volz
- **Frankfurt a. Main:** Czerny, Madelung
- **Giessen (?):** Hanle, Tschermak, Neumann
- **Heidelberg:** Bothe, Jensen
- **Marburg:** (Grüneisen), Walcher, Flügge, Vogt, C. Hermann
- **München:** (Sommerfeld), Gerlach, Bopp, Reich, E. Rüchardt
- **Würzburg:** Kuhlenkampff

b. **Institutes of Technology**
- **Darmstadt:** Vieweg, Scherzer
- **Karlsruhe:** Gerthsen
- **München:** (Zenneck), W. Meissner, Hettner, Klaus Schäfer
- **Stuttgart:** Regener, Fues, Dehlinger

c. **Research Institutes**
- **Max Planck-Institute**
  - für Metallforschung in Stuttgart: Köster, Glocker
  - für Medizinische Forschung in Heidelberg, Physikalische Abteilung: Bothe, Maier-Leibnitz
- **Kaiser Wilhelm-Institut** für physikalische Chemie in Berlin-Dahlem: Kallmann, Stranski

Within the last few months a new University, called "Freie Universität," was founded in the Western Sectors of Berlin.

II. British Zone and British Sector of Berlin

a. **Universities**
- **Bonn:** (Konen), Weizel
- **Göttingen:** Pohl, Kopfermann, R. Becker, Erwin Meyer, Houtermans, Masing, Bartels, Eucken
- **Hamburg:** Fleischmann, Bagge, Lenz, Harteck, Artmann
- **Kiel:** Lochte-Holtgreven, Unsöld
- **Köln:** Clemens Schäfer, Kirchner
- **Münster:** Kratzer, Seyffert
- **Technische Universität Berlin-Charlottenburg:** Ramsauer, Kallmann, Stranski

b. **Institutes of Technology**
- **Aachen:** Meixner, Fuchs
- **Braunschweig:** Justi, Cario (Diesselhorst)
- **Hannover:** Bartels, Braune
- **Bergakademie Clausthal-Zellerfeld:** (Valentiner), V. Auwers

c. **Research Institutes**
- **Max Planck-Institute für Physik in Göttingen:** Heisenberg, v. Laue, v. Weisäcker, Haxel, Wirtz
- **Max Planck-Institute**
  - für Strömungsforschung in Göttingen: (Prandtl), Betz, Tolmien, Vogelpohl
  - für physikalische Chemie in Göttingen: Bonhoeffer
  - für Eisenforschung in Düsseldorf: Wever

Otto Hahn in Göttingen is President of the Max-Planck-Gesellschaft.

III. French Zone

a. **Universities**
- **Freiburg i. Br:** (Mie), Gentner, Hönl, Trendelenburg
- **Mainz:** Klumb, Bechert, Strassmann
- **Tübingen:** Kossel

b. **Institutes of Technology**

c. **Research Institutes**
- **Kaiser Wilhelm-Institute**
  - für Chemie in Tailfingen, Kreis Balingen: (will be transferred to Mainz) Mattauch, Flammersfeld, Klemm, Waldmann
  - für Physik in Hechinger, Hohenzollern: Gentner, Schüler, Menzer, Molière
- **Kaiser-Wilhelm-Institut in Weissenau:** Regener
- **Research Institute of the Navy in St. Ludwig:** Schardin, Sauter, Trendelenburg, Max Kohler

IV. Soviet Zone and Soviet Sector of Berlin

a. **Universities**
- **Berlin:** Möglich, Rompe
- **Greifswald:** Seeliger
- **Halle:** Mönch
- **Jena:** Buchwald, Hund, Kersten
- **Rostock:** Kunze

b. **Institutes of Technology**
- **Dresden:** (Barkhausen)

c. **Research Institutes**
- **Astrophysikalisches Observatorium in Potsdam:** Kienle, Grotrian

Next let us consider the student body. The rush at the universities is terrific, for reasons arising from the war and identical with those

which have led to such a marked increase in the number of university students in the United States. But German universities admit only a small number from among the applicants. They are forced to introduce the ***numerus clausus***, which is fixed at about 3000–5000 at most universities. However, the University of München has 10,000 or more students. The German, as well as the occupation authorities, rigidly enforce its observance, and with good reason. To begin with, the available classrooms, laboratories, and equipment are insufficient for a higher number, and, furthermore, nobody quite knows in which academic professions a greater number of graduates could be suitably placed in years to come. But the selection of those acceptable among the flood of applicants imposes a heavy and exceptionally responsible task on professors. There is the constant danger that applicants of high ability and character may be rejected in favor of others who simply know how to create a good impression.

What the students live on is one of the mysteries of our times. Circumstances vary individually. The practice of earning a livelihood by doing odd jobs, previously unkown in Germany, now plays a limited part in helping students. Such jobs are so poorly paid, however, that they are hardly worth while. Some students were living on the remains of their savings, but the currency reform robbed them of that asset. Nevertheless, it seems that, in Göttingen at least, most of the students can carry on with their studies.

Naturally the universities try to ameliorate this situation; there are special midday and evening meals for students, an Academic Aid service, and various other means. I would mention that these organizations in turn have received ample support through packages from abroad. Please be assured that all kindhearted senders of such packages have won the highest gratitude of the recipients, and that this is perhaps the best type of peace propaganda. In view of the housing shortage the students, even the married ones, are limited to the minimum of living space, but of course they share this fate with their professors. Whoever has any available space at all is legally obliged to rent it out. The kitchen is usually shared by several families, and a professor's prerogative of a study of his own is by no means generally recognized.

The majority of universities and similar institutions have suffered heavy war damage. The Physics Institutes in most places are seriously damaged or even totally destroyed. Exceptions are the Universities of Erlangen, Göttingen, Heidelberg, and Tübingen, four cities that are practically unscathed. Likewise, the Physics Institutes of Würzburg, Marburg, and the Institute for Physical Chemistry in Hamburg show only minor damage and are able to carry on with their normal activities. Under what conditions, and in what quarters lectures on experimental physics are being conducted in other places can be determined only by investigation of individual cases. By now, most institutions of higher learning that suffered heavy war damage have probably erected temporary buildings for these purposes; part of the necessary labor was frequently contributed by students, who had to lend a hand in the task of rubble clearance and construction, this being in many cases a condition of their admittance to the universities. But even where the necessary space has been provided, there still is the very heavy problem of procuring equipment to illustrate the lectures. Since I myself live in Göttingen, which is unscathed, I have no idea how this difficulty is being tackled elsewhere.

The same difficulties apply also to the students' laboratory training and research work. And even where buildings and equipment are intact, the frequent cuts in electricity and gas supplies—and mind you, even the water supply fails at times—are a source of constant interruption of work. Therefore work is often done at night, when such disturbances are less frequent.

The procuring of new equipment has so far been difficult but possible within certain limits. For instance, in Heidelberg the cyclotron whose construction started during the war is operating now, and in Göttingen we have been using a betatron for several months. But since the currency reform, things have changed. While circumstances apparently vary from place to place, the Ministers of Finance have scaled down the budget of most universities to such an extent that the experimental laboratories can no longer carry on. If the Director of an Institute has 15 Deutschmarks per month at his disposal (this figure was quoted to me), while his expenditure for electricity alone is 100 D.M., the only course left open to him is to close the Institute.

I want to add that the budget of the Institute at Göttingen has not been appreciably curtailed. But I am still not convinced that the sums appropriated to the budget will actually be available. For instance, in some places the professors' salaries have not been paid in full or regularly since October.

Quite a different story is the fate of the Physical-Technical Reich Institute. During the war it split up into many small groups which spread out all over Germany and in the vicissitudes of war often changed their locations. Right now there is a fairly large group in Weida in Thüringen, in the Soviet Zone, which carries on under the name of "German Office for Measures and Weights" but no longer has the right to pursue research work. Further groups are located in Heidelberg, Göttingen, and various other places in the western zones. These are now destined to be welded into a Physical-Technical Institute in Brunswick which will resume the traditions of the Reich Institute. Its activities in the industrial sphere are supposed to be officially acceptable for all three western zones, and the three western military governments have given it recognition and support. In Brunswick well-built and intact laboratories with good workshops are available, but no physical apparatus and no library; and what is at least equally serious—there is no possibility at all of finding living accommodations for the roughly 70 families which the Institute has to resettle in Brunswick. Brunswick has suffered exceptionally heavy damage and, moreover, is overrun by refugees from the East. As a result, in the course of a whole year, the municipal administration was not able to put more than two apartments at the disposal of the Physical-Technical Institute, instead of the required 70. These figures are sufficient proof that without new construction of dwellings the Institute can never reconstitute itself.

Among the physical periodicals published at present, there are:

(*1*) *Die Annalen der Physik*, edited by Grueneisen and Moeglich, published by J. A. Barth, Leipzig.
(*2*) *Die Zeitschrift für Physik*, edited by von Pohl and v. Laue, published by Springer in Heidelberg and Göttingen.
(*3*) *Die Zeitschrift für Technische Physik*, edited by W. Meissner, published by Springer.
(*4*) *Die Naturwissenschaften*, edited by A. Eucken, published by Springer.
(*5*) A periodical entitled *Optik*, also a periodical on acoustics and a few others on specialized subjects.
(*6*) A new periodical, *Zeitschrift für Naturforschung*, edited by Klemm, published by Diederich in Wiesbaden.
(*7*) *Die Physikalischen Berichte*, edited by Dede and Schoen, published by Diederich.

Whether all these publications will survive the present plight remains to be seen. One must consider the fact that not even the German Institutes are able to subscribe to all these journals.

At any rate, there is no lack of subject matter. For instance, the *Zeitschrift für Physik*, which has just completed its first postwar volume, Bd. 124, has accumulated approximately 30 still unprinted manuscripts from the war period, and, in addition, about double that number of manuscripts submitted since then. We could not get them printed because the publishing firm failed to find a printing firm suitable for this work and capable of an adequate output. The previous handicap of a paper shortage is today no longer the greatest obstacle. The situation may be different as regards publications in the Soviet Zone.

Book production, too, labors under the same difficulties, and this assumes special significance in view of the fact that so many public and private libraries were seriously damaged by fire and looting, or in some instances even totally destroyed. A book trade, in the true meaning of the word, has not really existed since the war. Book stores have essentially become book exchanges, where one book can be exchanged for another. It is the student who suffers most severely from the shortage of textbooks. But at least Springer published a few volumes of Pohl's *Experimental Physics*, the Academic Publishing Company three volumes of lectures by Sommerfeld, and the second edition of my book *Roentgenstrahlinterferenzen*. But, unfortunately, book traffic across the zonal frontiers is sometimes handicapped. Volumes weighing more than one kilo, for instance, cannot be sent from Leipzig to the western zones as complete books, but have to be split up into sections which the recipient has to get reassembled. Likewise, the difficulty of making payments in other zones presents a serious obstacle.

We are almost completely isolated from the

publications of other countries. Of course the Military Governments have taken some measures to remedy this situation, but there is still much left to be done. I take this opportunity to address to everyone in this audience the sincere plea to make a contribution by sending as many reprints as possible to Germany.

Let my remaining words deal with the organization of German physicists. Right up to and even during the war the German Physical Society was in existence and celebrated its centenary in Berlin, in January 1945, notwithstanding the gravity of the general situation. The Control Commission for Germany dissolved this organization. But at least we were allowed to found new Physical Societies on a regional basis. Thus emerged the German Physical Society in the British Zone, of which I am Chairman; it was followed by an identical organization in Württemberg and Baden under the chairmanship of Regener; next came a Physical Society for Bavaria under the chairmanship of W. Meissner; and recently one for Hessen and one for the French Zone seem to have been sanctioned by the occupation authorities. There is no equivalent institution in the Soviet Zone. I ought to mention that we assume that the creation of a Western German State will pave the way for the unification of all the Physical Societies within its boundaries. So far these Societies have organized quite a few conventions. Two or three were held in Württemberg; in the British Zone there were four, three of them in Göttingen, and as recently as September one was held in Claustal. Despite all the difficulties caused by the currency reform, about 40 papers were given in Claustal for an audience of several hundred.

It is a gloomy picture I have to show you. Of course, the help has to come in the main from the German people themselves. Therefore the meeting in Claustal addressed the following plea to all Germans responsible for politics:

"Even before the war German science suffered great harm from lack of understanding by the then ruling government. War and its devastations have almost destroyed scientific life. After the war the scarcity of equipment and the difficult conditions of life have prevented or retarded reconstruction of the institutions. After the currency reform one would therefore have expected that the consolidation of the domestic economy would offer at least some moderate chance for work in the institutions. However, the rigorous curtailment of state expenditures has made any scientific work impossible.

"The physicists of all four zones, assembled in Claustal, together with their friends from England, Holland, Norway, and the U.S.A. lift their voices in warning in this extreme emergency.

"We do not need to point out the spiritual values which will be lost not only to our own country, but also to the whole world, if German science, which has given to mankind so much fundamental knowledge, is to be sacrificed. We would find it difficult to understand if these cultural values should be put aside with a regretful shrug of the shoulders, in favor of the more immediate necessities of life.

"But the matter at stake concerns not only cultural values, but also the economic existence and future of our people. It has become an undeniable truth, which one hesitates to repeat so often, that the research of today is the technique of tomorrow. Must we recall in vain that the industrial production of today is the fruit of past research and that research and science are an important part of the national assets, which must not be wasted?

"It seems impossible to stop the emigration of our best creative minds, because the possibilities of work in the scientific institutions of Germany are becoming almost nonexistent. It is, of course, tempting to save in times of need that which is not necessary to the immediate requirements of the day. This temptation will be further increased by the fact that the scholars (not many in numbers and living in the retired atmosphere of their work) are not accustomed to seek publicity and to call attention to themselves. However, the responsible leaders of the state should know that research is the seed of future industrial production and that it is economic suicide to save at this point. Compared to the total expenditures of the public funds, the amount assigned to research represents only a minimal fraction which is not in proportion to its importance. If the German people today decide that they cannot sacrifice a small portion of their national income for scientific research, the mass of the population will have to pay for this with a miserable standard of living for decades to come."

Force as a Basic Physical Quantity*

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IN considering how best to introduce the subject of physics using concepts and methods most in accord with the practical experience and background of students, it is well to clarify what is meant by the subject of physics. Physics may be specified as one type of description of the behavior of matter. In describing the behavior of matter, words and numbers are used. A word may be considered to be a symbol, either spoken or written, to represent an experience with matter. It can be pointed out that there is no way to obtain any information about matter except through the use of the senses. There are five senses which are easily recognized. We can hear, see, feel, smell, and taste and these senses are available for obtaining information about the properties of matter. This information consists of nothing more or less than sense experiences encountered in relation with matter. When it is desired to describe a relation between an observer and matter, certain words are used to denote sense experiences or combinations of sense experiences.

It may be agreed that it is possible to classify all words in any one language into two categories: those words which are defined and those words which are not defined, or those words which are definable and those which are not definable. A word which is definable is defined by a combination of other words. If one seeks to define each word that occurs in a definition, one must use in each new definition a set of new words that does not include the word originally defined. So, one may proceed on and on defining each word in a definition by a new combination of words, always avoiding the use of words previously defined until, finally, one arrives at a great many words which are themselves no longer definable. These nondefinable words can be recognized as words which denote simple, directly perceived sense experiences. As an example, consider time. Any attempt to define time in terms of other words is found to be useless. Much can be said about the measurement of time, or the comparison of one time interval with another time interval, but that does not give a definition of time. It is found necessary to accept time as something experienced and not defined. Time is, therefore, a name for a sense experience. Another such undefinable word is length. Length cannot be defined in terms of other words, but it can be experienced. Other undefinable words are hard, soft, red, green and similar qualities. The undefinable words are found to be words denoting directly perceived sense experiences.

In the study of the properties of matter there are found some sense experiences to which numbers can be assigned. Number may also be recognized as a sense experience. The experience of one object, two objects or many objects gives rise to the concept of number. The decimal system is generally recognized to have resulted because human beings have ten fingers. It is found possible to assign numbers to the experiences, length and time. To assign a number to a length, another length called a standard length is placed alongside the unknown length and the number of times required to obtain a match is counted. To assign a number to a time interval, the time interval is compared with the standard time intervals of a clock. Length and time thus have both a qualitative and a quantitative aspect. The qualitative aspect is the sense experience which, for a given length or time interval, will not be the same for any two persons. The sense of distance and the sense of time are very unreliable. The comparison between a given length or time interval and a standard length or time interval, however, will give the same number for any observer. By comparing an experience with a standard experience it is possible for an observer to describe an experience in a way to permit duplication by others. If two people measure a time interval and arrive at the same number, they are considered to have experienced the same time interval even though the time interval may seem to be very long to one and very short to the other.

With the undefinable concepts length and time

* Presented at the meeting of the Wisconsin Chapter of A.A.P.T., Eau Claire, Wisconsin, May 8, 1948.

as a basis it is possible to define several additional physical quantities such as: area, $A=d_1d_2$ where d_1 and d_2 are two lengths perpendicular to each other. Volume may be defined by $V=Ad$ where d is a length perpendicular to a surface of area A; velocity, $v=d/t$, where d is the distance covered in time t; and the average acceleration, $a=(v-u)/t$, where $(v-u)$ is the change in the velocity in time t. Area, volume, velocity and acceleration are sense experiences which can be described as combinations of the sense experiences, length and time. They have both qualitative and quantitative aspects. The qualitative aspect of velocity is the experience of distance and time combined. The quantitative aspect of velocity is the number obtained when the number for the distance is divided by the number for the time.

The choice as to which of the physical quantities, length, time, area, volume, velocity and acceleration, shall be considered as definable and which as undefinable is somewhat arbitrary. We might, for example, choose velocity and area as undefinable quantities and then define length as $l=A^{\frac{1}{2}}$ and time as $t=A^{\frac{1}{2}}/v$. These definitions would have just as much logical consistency as definitions using length and time as the undefinables. The choice of length and time as undefinable has been made subconsciously on the basis of convenience. The choice of length and time as undefinable gives a description of sense experiences which is easier to understand and use than the description which would have resulted with velocity and area as undefinable experiences. The use of length and time gives a description which is better suited to the way in which the human brain functions.

For the third basic quantity needed to describe mechanics, we may choose between force and mass. One of these must be considered as undefinable and the other defined in terms of it and length and time. Suppose we were to define force as $F=ma$, where m represents mass and a, acceleration. Acceleration has been defined in terms of the directly perceived and undefined sense experiences, length and time. Mass, however, is something which cannot affect the senses directly. Some physics books contain the statement that mass is inertia. If we try to define inertia, we might say inertia is a property by which an object resists acceleration. Acceleration has been defined. Resistance, however, turns out to be nothing more nor less than force. We have come back to the use of the word *force* which we set out to define: It is thus recognized that force is a directly perceived sense experience, while mass is not.

The mass of an object may be perceived only through application of a force and observation of the acceleration which results. The comparison of masses of two objects from the change in motion which results from a collision between them depends on the application of forces of equal magnitude to the objects.

Another difficulty with trying to define force by the equation $F=ma$ is that there are many forces which do not produce acceleration. Forces on an object held in equilibrium are real and definite although no acceleration results. Defining force by the equality, $F=ma$, introduces confusion in describing forces in equilibrium. Since an attempt to define force in this way leads to a definition of force in terms of itself and introduces confusion into the description of forces in equilibrium, it is more satisfactory to use Newton's second law in the form $m=F/a$ to give a definition of mass. Force can be recognized as a directly perceived and undefinable sense experience like length and time.

TABLE I. Units of force and mass.

Force	pound	gram	poundal	dyne	newton
Mass	slug	gravitational gram	pound	gram	kilogram

The units of force and mass are often a point of confusion for students. Force may be measured in grams, pounds, dynes, poundals and newtons. For each of these units of force a corresponding unit of mass is needed. Table I shows these units. The slug and gravitational gram are not usually mentioned in textbooks. They are, however, necessary to preserve consistency in the development of physics.

The description of physics in terms of force, distance and time fits the past experience of students. Students are acquainted with these quantities before beginning the study of physics but they have no conception of mass. Students know what force is and it is not hard for them to recognize that for every force on one object, there

is an opposite force equal in magnitude on another object. Newton's third law may be most conveniently written as $F_1 = -F_2$.

Students can also understand the statement that when the product of force and distance (with force parallel to distance) is measured, it is discovered that the total sum of such products in any experiment is a constant. It can be stated that what is measured by the product Fs cannot be created and cannot be destroyed; it can, however, be stored in an object or passed from one object to another. For convenience, the product of force times distance is called work. When work is stored in an object it is called energy. Energy may, therefore, be defined as stored work. Thus the student's vague concepts of work and energy may be made exact and understandable in terms of length, time, and force.

It may be considered by some that mass is better suited than force to serve as a basic quantity of physics because it is easier to preserve a standard of mass than a standard of force. The preservation of a standard is only one of five necessary specifications of any physical quantity. These are (1) the name of the physical quantity, (2) the definition of the quantity, (3) the name of the unit in which the quantity is to be measured, (4) the definition of the unit and (5) the method for preserving a standard unit. The definition of a quantity can be quite different from the method for preserving a standard unit for the quantity.

Whether mass is considered as defined by $m = F/a$ or considered as an undefined concept is immaterial in arranging to preserve a standard for mass. If, however, it is desired to preserve a standard of force instead of a standard of mass it can be done by specifying one dyne as that force which will give to the standard object known by name as the standard kilogram an acceleration of $1/1000$ cm/sec^2.

For rotational motion, the three quantities, angle, torque, and time, take the place of length, force, and time. The resulting definitions and equations for rotation are identical with the equations and definitions for translation if rotational inertia is defined as $I = T/\alpha$, where T means resultant torque and α denotes angular acceleration. There is obtained a complete correspondence between the equations of translation and rotation with rotational inertia taking the place of mass. All equations for rotation and translation are identical except for replacement of translational physical quantities by rotational physical quantities. Newton's laws of motion can be stated for rotation in the same way as for translation: (1) An object remains permanently at rest, or in uniform rotation, unless subjected to a unbalanced torque, (2) the resultant torque on an object is equal to the rotational inertia times angular acceleration and (3) for every torque on one object there is a torque equal in magnitude and opposite in direction on another object. If these laws are described by equations, they take the forms: (1) if $T=0$, $\alpha=0$ (2) $T = I\alpha$ and (3) $T_1 = -T_2$. Newton's third law of motion for rotational motion has not been included in most elementary textbooks and it is sometimes difficult for students to see where the torque equal in magnitude and opposite in direction comes in. For a wheel mounted on a wall with a weight hung on a rope wound around the axle to cause the wheel to rotate with angular acceleration, it is clear there is a torque applied to the wheel. The equal and opposite torque is a torque applied to the earth. The force of attraction between the weight and the center of the earth pulls up on the earth and this with the push of the axle down on the earth gives to the earth a torque which is equal and opposite to the torque on the wheel.

In conclusion, it may be said that since physics is a description of sense experiences, the use of the experience *force* instead of the concept *mass* as one of the basic quantities in terms of which other quantities are defined, leads to a description more closely related to our senses, and therefore, more sensible than a description in terms of mass as a basic quantity.

A man who has committed a mistake and doesn't correct it is committing another mistake.—CONFUCIUS.

Caustic Curves by Geometric Constructio.

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A DISCUSSION of the caustics by reflection in a concave spherical reflecting surface appeared recently in this journal.[1] As an alternative to the analytical method of GFH, we present here a geometrical construction which yields points on such caustics and can be extended to the caustics produced by the other conic sections as well. This construction has already found application in studying conjugate foci of mirrors of circular cross section used in x-ray image formation.[2]

Let us consider a spherical concave reflector with radius r and center at C (Fig. 1). We shall deal only with meridian rays diverging from a point P and hence we draw only the meridian circle in the plane of the paper. Let M be an arbitrary point on the surface. Following GFH, let u represent the length PM of the incident ray making an angle i with the normal CM. This ray is reflected at M. Similarly the ray PM' at a nearby point M' is incident at an angle $i+di$ with the normal CM'. The rays reflected at M and M' will in general intersect, sometimes converging to a point S as is drawn in the figure, and sometimes diverging from a virtual point behind the mirror. As the construction is entirely general it is sufficient to deal only with the convergent case. Let w stand for the length MS. Now S is not on the caustic, but if we let M' approach M, that is, if we let $di\rightarrow 0$, then S approaches a limiting position S' and w approaches a limiting length w'. In this limiting case GFH shows that

$$\frac{1}{u}+\frac{1}{w'}=\frac{2}{r\cos i}\equiv\frac{1}{f}. \tag{1}$$

We shall use w' to indicate that particular value of w which satisfies Eq. (1). In the general case where di has not been allowed to approach zero, GFH shows that the mirror equation is more complicated. The point S' defined above lies on the caustic and we propose to determine its location by a purely geometric construction. We shall describe the construction first and then prove that it is in agreement with Eq. (1).

(*1*) *Geometric Construction.*—In Fig. 2 let the circle of radius $CM=r$ be the mirror circle, and the point P be the source of radiation. Draw the Rowland circle whose diameter is CM, and let A be the point of intersection of PM, the incident ray, and this circle. A circular arc of radius MA with center at M intersects the Rowland circle at B and MB is obviously the direction of the reflected ray. Draw the line AB and let it intersect CM at U (for "useful"). Draw the line PU and extend it to meet MB at S''. If we denote PM by u then MS'' equals w' as defined by Eq. (1), and consequently S'' coincides with S' and is a point on the caustic.

(*2*) *Proof.*—Let N be the mid-point of AM. Since $CM=r$, it is apparent from a consideration of triangle MAC that $MA=r\cos i$. Hence $AN=NM=(r\cos i)/2=f$. Triangle ANU can be proved to be isosceles, so that $NU=f$. Similarly, triangle NUM is isosceles, and angle $NUM=i$, making NU parallel to MB. In the similar triangles PNU and PMS'' we have $MS''/NU=PM/(PM-NM)$. The relations $PM=u$ and $NU=NM=f$ are now used to obtain

$$\frac{1}{u}+\frac{1}{MS''}=\frac{1}{f}=\frac{2}{r\cos i}.$$

Hence $MS''=w'$. We have proved that S'' lies on the caustic. Incidentally, it is easy to prove that the length $MQ=v$ obtained by letting BM meet CP at Q obeys the relations

$$\frac{1}{u}-\frac{1}{v}=\frac{1}{f'}=\frac{2\cos i}{r}$$

in accordance with GFH, except that in our notation all quantities are positive. When PC/r is almost unity, as it is in cases of x-ray reflection, the sagittal rays produce a virtual image. Since the points P and M were chosen arbitrarily, we have exhibited a general method of

[1] G. F. Herrenden-Harker, *Am. J. Physics* **16**, 272 (1948), hereafter referred to as GFH.
[2] Paul Kirkpatrick and A. V. Baez, *J. Opt. Soc. Am.* **38**, 766 (1948).

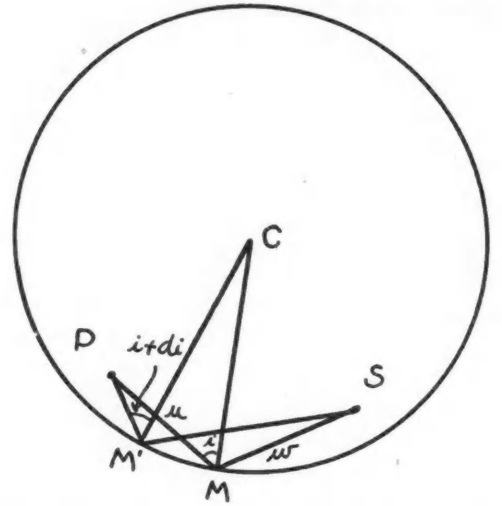

FIG. 1. Meridian rays leaving P are reflected to meet at S.

constructing all points on the caustic. We need simply consider the rays emanating from P (considered fixed) and hitting a variable point M on the mirror. Figure 3 illustrates the caustic produced by this construction for the case in which P lies on the mirror circle. (Compare GFH, Fig. 9.) Figure 4 gives the caustic when $PC/r=0.7$. (Compare GFH, Fig. 10.) It exhibits a virtual part of the caustic not shown in GFH.

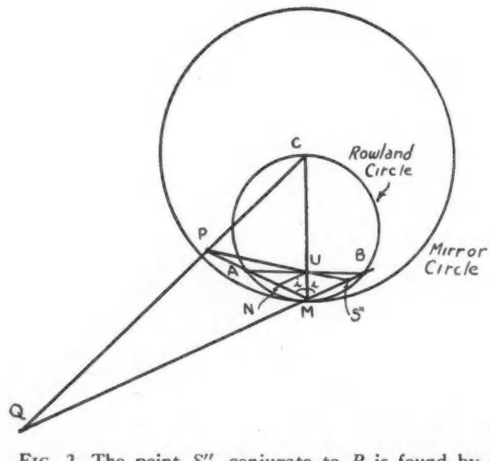

FIG. 2. The point S'', conjugate to P is found by a geometric construction. S'' is a real image of P formed by meridian rays and Q is a virtual image formed by sagittal rays.

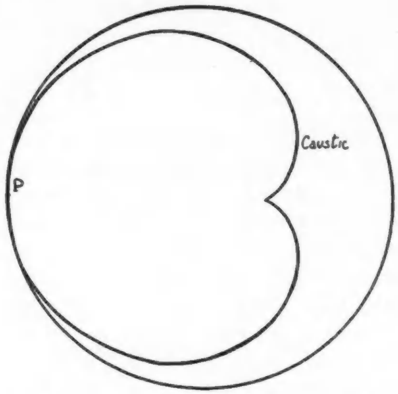

FIG. 3. The points on this caustic were found by geometric construction.

(*3*) *Nomograph.*—In Fig. 2, if $i=60°$, then $MU=\frac{1}{2}AM=f$ so that the following nomograph for solving $1/u+1/v=1/f$ suggests itself.[3]

Construct an angle PMS of 120° and its bisector CM. PM is an arbitrary length u and MS is an arbitrary length v. The point U is the intersection of PS and CM, and by our previous considerations MU obviously represents that value of f which satisfies $1/u+1/v=1/f$. Thus U is clearly the useful point of our first construction. Given any two of the quantities u, v and f, the third can be found by obvious modifications of the method just given. This nomograph is extremely useful in visualizing geometrically the effect of varying any two of the quantities u, v and f, while leaving the other fixed.

(*4*) *Geometric Construction for the center of curvature of an ellipse.*—It is curious that a purely geometric theorem results from a consideration of the optical properties of the conics. In Fig. 5, F_1 and F_2 are the foci of the ellipse

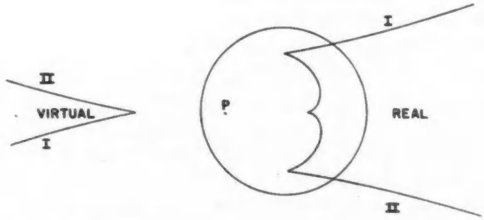

FIG. 4. This caustic exhibits a virtual region also found by geometric construction.

[3] We found that this special case of our construction appears in textbooks. See, for example, S. Chapman, *Laboratory manual, light and heat* (National Press, Millbrae, Calif., 1948), p. 6.

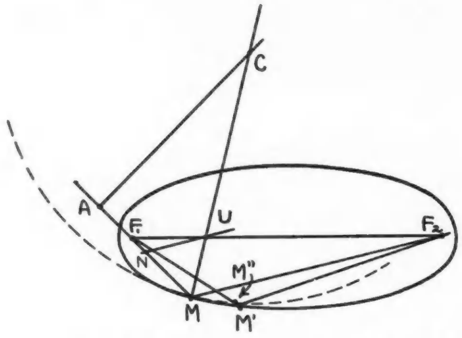

FIG. 5. Geometric construction for the center of curvature of an ellipse.

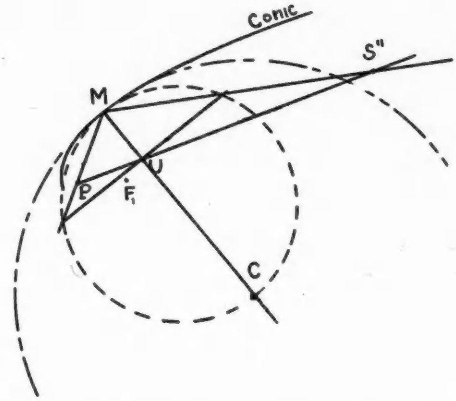

FIG. 6. M is an arbitrary point on a conic other than a circle. S'' is a point on the caustic produced by the conic when light emanates from a point source at P.

shown; M and M' are points on the ellipse. The rays F_1M and F_1M' meet at F_2 after reflection, due to the focal property of the ellipse. Now imagine the osculating circle at M acting as a reflector. It would reflect the ray F_1M in the direction MF_2 but another ray with the direction of F_1M' would not go to F_2 after reflection from the circle. Let M'' be the intersection of the osculating circle and the line F_1M'. If M'' is infinitesimally close to M, the ray reflected at M'' will pass infinitesimally close to F_2 and in the limit, as $M''\rightarrow M$, F_2 must be the point associated with M on the caustic due to reflection by the osculating circle. To construct the osculating circle at M, then, we treat the foci F_1 and F_2 as P and S'', respectively, of Fig. 2. The validity of the following instructions can be traced by the reader to the construction given earlier.

Draw the line which bisects angle F_1MF_2 (Fig. 5). This is the normal to the ellipse at M and contains the center of curvature. Let it meet F_1F_2 at U. Through U draw a line parallel to MF_2 meeting F_1M at N. The parallelism suggests that $NM=f$ when $F_1M=u$ and $MF_2=w'$. Extend MN to A so that $MA=2MN=2f$. At A construct a perpendicular to MA and let it meet MU at C.

The lettering corresponds to that of Fig. 2 with F_1 and F_2 replacing P and S'', respectively. In Fig. 2 it can be seen that CA is perpendicular to AM. In Fig. 5, then, C is the center of curvature of the ellipse at M. As the other conics have focal properties similar to the focal property of the ellipse, this method with only slight modifications can be used to find the center of curvature of any conic.

(5) *Caustics by conics other than a circle.*—We shall merely indicate how, knowing the locus of the centers of curvature of a curve, we can draw the caustic that corresponds to it. In discussing the conics, we must put our source of radiation away from a focus, otherwise the caustic is degenerate. Thus, in Fig. 6, P is a point source of radiation and M is an arbitrary point on the conic whose focal point is F_1. By the method just described we find the center of curvature C of the conic at M and draw the osculating circle of radius CM. Treating this as a mirror circle, and considering reflections from an infinitesimally small segment at M, we find the point S'' conjugate to P by the first construction given above. This entails drawing the Rowland circle of diameter CM, etc. Thus S'' is a point on the caustic which we seek. Other points are found by a repetition of this process.

There is no adequate defense, except stupidity, against the impact of a new idea.—P. W. BRIDGMAN.

Penetrating Showers in Lead*

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MOST of the cosmic rays observed at sea level are fast penetrating particles whose properties are fairly well known. They have unit charge, mass about 210 electron masses and are unstable, with a half-life of 2.15 microseconds measured when the particles are brought to rest. They are called either mesons or mesotrons, depending on the observer. The instability of these particles and their short half-life make it certain that they are secondaries produced in our atmosphere. It is postulated that they are produced in collisions of the primary cosmic rays with the nuclei in the upper atmosphere, and the details of this nuclear interaction are of very great interest to those investigating nuclear forces. It would be desirable to conduct experiments on these production processes right where most of them occur —namely, in the top one-tenth of the atmosphere—and indeed, some experiments have been carried out in that region by means of balloon and rocket flights.

It is known, however, that production of penetrating particles occurs also at lower altitudes, and, in particular, it is possible to investigate these nuclear interactions at sea level where, although the events are rare, the observation time is limited only by the patience of the observer and not by the detecting equipment.

The term "penetrating particle" is defined in two ways, depending on the equipment used to detect it. In a Geiger counter experiment, a penetrating particle must go through 10 to 20 cm of lead (the latter figure being safer to use) before tripping a counter in order that it may be identified as a penetrating particle. Fast electrons cannot penetrate large thicknesses of lead because they dissipate their energy very rapidly in making an electron shower. In a cloud chamber, however, if a particle goes through only 2 cm or 3 cm of lead without making secondary particles such as would be made by an electron under similar circumstances, it is safe to classify it as a penetrating particle. By these definitions, penetrating particles are not necessarily mesons; they may be any particle with unit charge, and mass anything greater than 5 to 10 times the electron mass. Thus fast protons are not excluded. We also define a "penetrating shower" as one where two or more penetrating particles are time-coincident in the vicinity of the detecting apparatus.

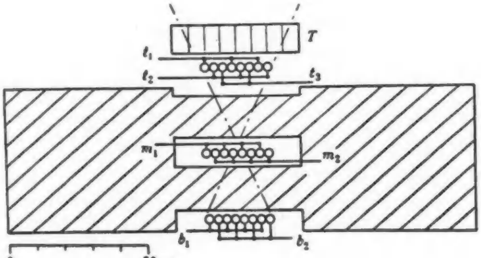

FIG. 1. Counter set for detection of penetrating showers (Jánossy).

Experimental Methods

It has been implied that cloud chambers and Geiger counters are the principal means of detecting penetrating showers. Ionization chambers have also been used, in conjunction with Geiger counters.

In counter experiments it is essential that large thicknesses of lead be used so that penetrating particles are selected, and that some arrangement of counters be used that detects showers of particles rather than single particles. The latter is accomplished by using horizontal trays of counters with alternate counters in coincidence so that at least two particles must be present for a count to be registered.

The most comprehensive counter experiments on penetrating showers at sea level have been made by Jánossy and his co-workers. A typical arrangement of counters and lead used by Jánossy and Broadbent[1] is shown in Fig. 1. In order to register a count, a sevenfold coincidence

* Paper presented at the Pasadena meeting of the American Physical Society, June 24, 1948.

[1] Broadbent and Jánossy, *Proc. Roy. Soc.* **A190**, 497 (1947).

of counter trays $t_1t_2t_3$, m_1m_2, and b_1b_2 was required. The absorber T was at the disposal of the experimenters. In order to separate the effects due to extensive electron showers, an unshielded tray E of counters (not shown in Fig. 1) was placed at a horizontal distance of a few meters from the penetrating shower set P, and records were made of whether or not the tray E registered a count simultaneously with set P. By this means Jánossy distinguished *local* penetrating showers and *extensive* penetrating showers.

By varying the thickness and material of absorber T it was possible to discover some properties of the particles in the penetrating showers. The local penetrating showers showed an increase when material was placed over the counter set, as shown in Fig. 2. This was interpreted as due to production of penetrating particles in T by primaries of some kind. The mean free path of the primaries for this nuclear interaction was found from the curve of Fig. 2 to be of the order of 5 cm of lead, and the fact that the rate of penetrating showers is still increasing with 10 cm of lead over the counter set indicates that photons or electrons are not the primary particles. Very few photons or electrons penetrate 10 cm of lead. It is now generally believed that fast protons or neutrons are the primaries for these events.

Thus we are observing the results of a nuclear interaction between fast nucleons and the lead nuclei, giving rise to the penetrating *particles* which trip the counters imbedded in the lead. A curve that shows the frequency of production of penetrating particles as a function of thickness of material is known as a *transition curve.*

A significant piece of evidence in favor of this hypothesis is that the transition curves for lead are identical within the experimental error with those for paraffin, when the thickness of the absorber is calculated in g/cm². Thus the effect is mass-proportional.

In the alternative case of *extensive* penetrating showers, that is, when the penetrating shower observed is in the midst of a large electron shower, the transition effect for lead is similar to the electron transition effect; it shows a maximum at about 2 cm of lead. Paraffin shows no transition effect. Clearly the electrons present have an effect on the counters which is related in some way to the penetrating component, but the nature of the relationship and the nature of the penetrating particles is not clear from Jánossy's experiments.

To investigate the properties of the particles in the penetrating showers, Jánossy and Rochester[2] installed a cloud chamber in the center of the apparatus and modified the counter arrangement to make sure that some of the penetrating particles would go through the cloud chamber. Across the middle of the cloud chamber they placed a lead plate 2.3 cm in thickness, and classified as *penetrating* any particle which passed through this plate without multiplication or large-angle scattering. The pictures taken served to emphasize the complexity of penetrating showers; not only penetrating particles but heavy particles were found, and, even more important, high energy electrons and photons were found in the penetrating showers. The thicknesses of lead were so great that these electrons could not have come from the outside, but must have been made in the lead, perhaps at the same time as the penetrating particles.

The cloud chamber used by Jánossy and Rochester had the additional advantage that some further identification of the particles could be made. Using a somewhat different arrangement, with the cloud chamber in a magnetic field, Rochester, Butler and Runcorn[3] have identified as a meson a particle produced in a lead plate by a component of a penetrating shower.

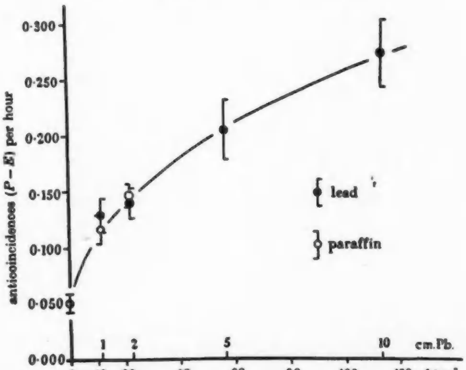

FIG. 2. Transition effect for local penetrating showers (Jánossy).

[2] Rochester, *Proc. Roy. Soc.* **A187**, 464 (1946).
[3] Rochester, Butler, and Runcorn, *Nature* **159**, 227 (1947).

Multiplate Cloud Chambers

Another tool for the investigation of penetrating showers has been the multiplate cloud chamber. In these chambers many lead plates are placed horizontally, separated by distances of about an inch. Thus the length of track available for observation between the plates is not very long, but is usually sufficient to indicate the direction of motion of the particle and the relative ionization. Multiplate cloud chambers have the advantage that the progress of the shower can be observed in various stages. It is possible in favorable cases to make identification of particles. Finally, the cloud chamber can be made very large, and thicknesses of lead comparable to the mean free path of nucleons in lead are fairly easy to obtain.

A cloud chamber now in use at the University of California[4] contains 16 one-half inch thick slabs of lead—a total of over 20 cm thickness. With this chamber it is possible to make detailed observations of the production of penetrating particles, the angular distribution, the multiplicity, the scattering, and the simultaneous production of electronic radiation.

The cloud chamber is counter controlled, with a very loose selection of penetrating showers. The counter arrangement is shown schematically in Fig. 3. A fourfold coincidence is required to trip

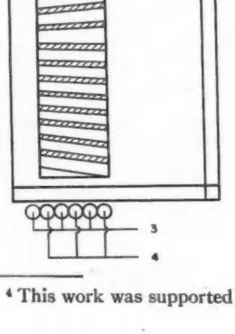

FIG. 3. Side view of cloud chamber and counter apparatus. The lead plates slope so as to converge at the camera lens.

[4] This work was supported in part by ONR.

the cloud chamber, at least one particle above and at least two below. The result is that most of the pictures at sea level show a single penetrating particle which makes a knock-on electron near the bottom of the chamber so that two counters are tripped. Occasionally, however, a genuine penetrating shower occurs in the chamber. Examples of these are shown in Figs. 4 to 7. To illustrate the analysis of one of these penetrating showers, let us consider the shower of Fig. 4 in detail.

(1) The particle that produces the shower goes through the first three plates without interaction. It is therefore classed as a penetrating particle.

(2) Under the fourth plate its ionization becomes noticeably heavier. This is often a sign that the particle is slowing down, but it probably is not in this case; only a very fast particle could produce the shower below. The other possibility is that the original particle has made another in the fourth plate and the two come out together, giving the impression of heavier ionization.

(3) In the fifth plate a *star* is created; two heavily ionizing particles appear. The heavier of these appears to make two penetrating particles; it may be a fast α-particle.

(4) Below the eighth plate it becomes clear that there are two penetrating particles very slightly diverging. These may be the two particles observed under the fourth plate (see paragraph 2).

(5) In the tenth plate a typical penetrating shower occurs. Penetrating particles (five of them clearly discernible) are projected at various angles from the original track. The wide angular dispersion of penetrating particles is characteristic of these showers.

(6) Directly in line with the original particle, an electron shower can be seen, the particles of which penetrate at least five plates. Simultaneous production of electronic radiation is often observed in these showers.

(7) One of the penetrating particles produced in the tenth plate slows down, becomes heavily ionizing and stops (in the nick of time) in the 16th plate. It is identified as a meson because of its heavy ionization near the end of its range. The other penetrating particles cannot be identified.

(8) The distance between successive productions of penetrating particles—in this case three inches of lead—is the free path of the primary particle in lead. When large numbers of showers are observed which exhibit successive events, the mean free path can be obtained.

At sea level a picture of a penetrating shower is obtained on the average with the arrangement shown, about once in two days, so that the total number of penetrating showers to be analyzed is not large. The results of the analysis answer some

FIG. 4. Local penetrating shower with successive production of penetrating particles.

FIG. 5. A penetrating particle in an air shower makes a penetrating shower containing high energy electrons.

FIG. 6. A penetrating particle makes a penetrating shower containing no high energy electrons.

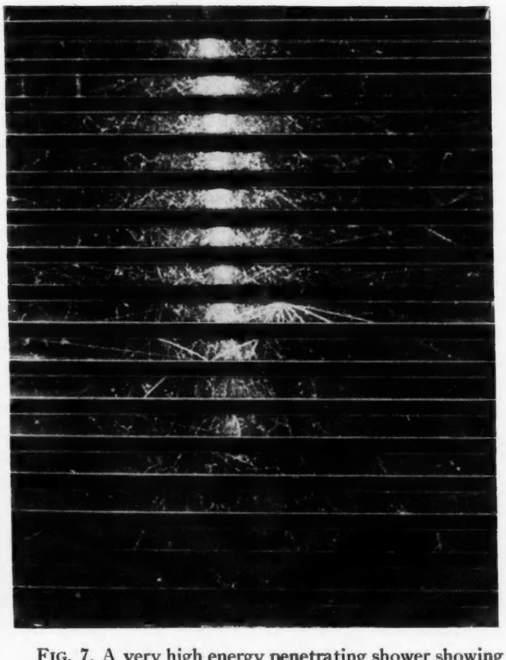

FIG. 7. A very high energy penetrating shower showing production of high energy penetrating particles, a large electron shower and a large nuclear explosion.

questions raised by the counter experiments, but pose additional questions.

The pictures clearly show:

(*1*) *Simultaneous multiple production of penetrating particles, some of which can be identified as mesons.*—One can infer that this type of process also occurs at the top of the atmosphere, the results of which are observed at sea level as the penetrating (meson) component of cosmic rays.

(*2*) *Successive multiple production of penetrating particles.*—This means either that the primary particle is not catastrophically absorbed in the first event and can continue to make further events or that particles are produced in the first event which are capable of producing additional nuclear events. These two possibilities are not, of course, exclusive.

(*3*) *Simultaneous production of penetrating particles and electronic radiation.*—Since this phenomenon occurs, it may be possible to describe all the important components of cosmic radiation, including even the very large and extensive air showers, in terms of interactions of primary protons with air nuclei in the upper atmosphere. Primary electrons are no longer thought to be required to produce electronic radiation, which is indeed fortunate since Rossi[5] has recently shown that the percentage of electrons or photons in the primary radiation is extremely small.

The question arises as to the mechanism of production of electronic radiation as a result of collisions. The meson theorists answer that there may be very short-lived mesons produced in the collision, possibly neutral mesons which decay rapidly into pairs of γ-rays of high energy which then create the meson shower. This question is far from settled, and new varieties of mesons are invented almost every day to account for the observed phenomena.

A second question is: what kind of charged mesons are produced in the interaction and are these the ones we observe at sea level? Observations with photographic emulsions have shown that two kinds of charged mesons exist: π-mesons which are created in nuclear reactions, and μ-mesons, the decay products of π-mesons, which are presumably those we observe at sea level. These two types differ only slightly in mass,[6] but are very different in their properties of nuclear interaction. If mesons are present in the shower, they might exhibit anomalous nuclear scattering or some other kind of nuclear interaction. Perhaps some of the successive events are caused by interactions due to π-mesons. It is not likely that very many of the initial events are caused by π-mesons, since their short life (approximately 10^{-8} sec) makes them rather rare at sea level.

It is very likely that fast protons (which would look like penetrating particles) and fast neutrons are created in penetrating showers. These fast nucleons, if energetic enough, can create further nuclear events, and in particular the neutrons produced are probably responsible for many of the stars observed in photographic plates and in cloud chambers.

The extensive penetrating air showers may be explained as well-developed, high energy penetrating showers. It is known from counter experiments by Cocconi[7] and by Treat and Greisen[8] that the electron showers accompany extensive penetrating showers and vice versa. Cloud-chamber pictures of these extensive penetrating showers sometimes show local production of penetrating particles, usually by penetrating particles already present in the extensive shower. Some combination of these complicated events must have occurred for every count observed by Jánossy in his investigation of extensive penetrating showers. An example of this kind is shown in Fig. 5.

A phenomenon of particular interest, still completely unexplained, is the nearly complete disruption of a nucleus, such as is shown in Fig. 7, just to the right of the main shower. Such disruptions have also been observed in photographic emulsions in which a large number of particles emerged from a single point. Such a process as this requires a large amount of energy, and one can postulate that a very fast particle might have hit the nucleus. In this case, however, the particle goes through the nucleus so fast that it has time to interact with only one or two nucleons which may, indeed, be ejected violently. The other particles, however, have been nearly unaffected by

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[5] Hulsizer and Rossi, *Physical Rev.* **73**, 1402 (1948).

[6] Latest values of masses are: π-meson, 285 electron masses; μ-meson 215 electron masses. (Private communication from R. B. Brode and C. M. G. Lattes.)

[7] Cocconi, Loverdo, and Tongiorgi, *Physical Rev.* **70**, 852 (1946).

[8] Treat and Greisen, *Physical Rev.* **74**, 414 (1948).

the passage of the fast particle. In the photograph shown, we have a case of many *heavy* particles produced, and no fast penetrating particles. To explain such a complete disruption, one may have to postulate a collision with a much larger particle (not likely in the case shown where no heavy track appears above) or a collision with an antiproton or an antineutron or an ordinary proton or neutron in an abnormal energy state. An "antiproton" is related to a proton as a positron is to an electron. The hypothetical antiproton and an ordinary proton would annihilate each other on collision, releasing a large amount of energy. Such particles have been suggested to have transitory *virtual* existence; perhaps one was caught here before it could return to its normal state.

Reproductions of Prints, Drawings and Paintings of Interest in the History of Physics

41. *Vanity Fair* Caricatures of Charles Darwin and Thomas Huxley

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THE question may well be asked why caricatures of CHARLES DARWIN (1809–1882) and THOMAS HUXLEY (1825–1895) should appear in a series of historical reproductions of special interest in physics. The reasons are that their names, together with that of JOHN TYNDALL, whose caricature has already appeared in this series, are "inseparably connected with the battle which began in the middle of the 19th century for making the new standpoint of modern science part of the accepted philosophy in general life," and that these caricatures together with the written account that accompanied them show quite clearly the prestige that science acquired during the Victorian era.

These caricatures were originally published in *Vanity Fair*[1] for 1871. The following statements accompanied them:

Mr. Charles R. Darwin

"In all times, and among all peoples, whenever and wherever the faculty of thought has existed, men have asked the question: 'How come we here?' and there is scarcely a form of society but has produced one or more kinds of answer to the inquiry. Up to within the last hundred years these answers have one and all been presented in the form of a Revelation from an Authority entirely outside the men and the world to whom and to which the matter relates. The fables that have thus been presented to mankind have been of the most various and the most conflicting character, and have been referred to the sanction of the most dissimilar Authorities. They have agreed in one only respect—that they have all been of the most childish invention, and that the elaboration of the systems built up thereupon has been of the very rudest kind.

"While the vulgar many of all countries have universally received the local Fable as it was presented, the stronger-headed few have as invariably rejected it. For hundred of years, as we know, and as we may infer, for thousands, the few were fain to content themselves with the conclusion that they knew nothing whatever about the earth and its inhabitants. In modern times, however, the method has been adopted of interrogating that earth and those inhabitants themselves without reference to any real or supposed external Authority; and although in so vast a field of labour it has as yet not been possible to achieve any very great results, certain new theories have been built up of which it can at least be said that they are more presentable than any of the old fables.

"Among these theories one of the most striking is that which Mr. Darwin has given to the world with reference to the Origin of Species by means of Natural Selection. Mr. Darwin, who was born sixty-two years ago, has spent the whole of a most

[1] See Reproduction No. 40 in this series, *Am. J. Physics* 17, 86 (1949).

PLATE 1. "Natural Selection." [From *Vanity Fair*, September 30, 1871.]

PLATE 2. "A Great Med'cine-Man among the Inqui-ring Redskins." [From *Vanity Fair*, January 28, 1871.]

laborious life in close converse with the material world in which we live, and the beings that it has from time to time seen upon its surface. He has thus become one of the most accomplished naturalists now in existence, and any theoretical structure that he builds upon his researches must be regarded with great respect. His books are written to a large extent for and appeal to ordinary men. This, indeed, it is which gives them their great importance. This, however, makes it also allowable to say that to ordinary men the chain of inferences seems to be very loosely hung together by which he seeks to establish that the various species of animals now existing on the earth inherit all their immense dissimilarities from a common ancestor, and that they have acquired their wide differences of development simply from individual aberrations. Nevertheless, so unknown to us are our fellow-beings that even for ordinary men his writings have all the charm of romances; while they will remain to all time, if nothing else, at least a record of earnest and honest devotion to the solution of the most momentous of the problems by which mankind are surrounded."

Professor Huxley

"PROFESSOR HUXLEY, the inventor of Protoplasm, is a great Med'cine Man among the Inquiring Redskins. The renowned Ongpatonga himself was not more popular in the solemn Calumet dance than Professor Huxley in the annual gatherings and other ceremonials observed by the various tribes of the great Philistine family who roam over the deserts of the metropolis, to the terror of the ecclesiastical police and the intense disgust of the respectable portion of society who go clothed and in their right mind. Professor Huxley, like the rest of the Ongpatonga tribe, is

wonderfully matter-of-fact; but, with all his hardness and anti-transcendentalism, his geniality of temperament and his happy talent for illustration are hardly less remarkable than the logical clearness of his discourse—of which it will be quite sufficient to state that the denizens of *Vanity Fair* will find a good popular specimen in his "Lay Sermons." Professor Huxley favours the movement for the Scientific Education of Women. He wants them to be the associates of men in the 'feast of reason' and 'the flow of soul,' and would no longer feed them with the fag-ends and scraps of knowledge which they have been accustomed to pick up. In this respect his practice differs essentially from that of the Un-inqui-ring Redskins, whose squaws are compelled to keep in the background until their lords have dined, and are then admitted to a scramble for the bones and shreds of the repast. If Ongpatonga has a fault, it is one which may fairly be ascribed to incompleteness rather than intellectual vice. He refuses to believe in angels, because the telescope has not yet discovered them. Like a man who hops on one leg, instead of walking erect with his face heavenwards, he has to pick his steps with care through the mud of Materialism, and in this way it has come to pass that he has stumbled on Protoplasm, which he sees on the seamy side, but not on the shining inner surface. In good time, when he is tired of hopping, he will get both his feet firmly on the ground, and then, trusting to his eyes and his inner senses, he will have more to tell the world than the telescope has ever told him. Take him for all in all, there is no popular teacher who has contributed more to the awakening of the intellect, and whose career in the future may be more confidently associated in idea with all that is manly and progressive in social science, and comprehensive, to say the least, in physical research."

This brings me to my next point, one that has often been stressed by other students of this subject. It is the extreme difficulty of giving away scientific secrets. I have never tried to do it, so I have no first-hand knowledge in this context. But I should imagine it would be rather like teaching. All of us have experienced the teaching process as receivers and some of us have also tried to serve on the transmitting end. Of course, if the secrecy goes so far as to include the mere fact of the existence of a project on a certain subject, such a secret can be given away without difficulty. But the amount of essential detail even with regard to principles and especially with regard to specific designs, that inheres in any modern scientific military device is fantastically great. To give away such secrets one would have to transfer vast quantities of drawings and documents. Even those are usually so unclear without explanation that the receiver would need to be given a special course of instruction in their meaning. Even this, to be really effective, requires the receiver to be a man of high scientific and technical training.—E. U. CONDON (1948).

I know nothing more deadening to original ideas than keeping a man's nose firmly fixed to the grindstone. Even directors need a change, and young men should have opportunities of meeting other young men working in other parts of the country. Ideas are more likely to come from such meetings with colleagues than by holding men down to some work in which there might be no progress at all. No laboratory today is self-sufficing.—LORD RUTHERFORD.

Physics in Premedical Education

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PLANS for the improvement of premedical training in physics have, in recent years, been discussed frequently and from various points of view. It has been pointed out that medicine is becoming more and more scientific, to a degree which makes a sound training in the basic sciences urgently necessary for every medical man.[1,2] Regret has been expressed that many physics courses attended by premedical students neglect the needs of the future students of medicine, or biology, by taking a one-sided orientation from the field of engineering; to remedy this situation a demand has been made for an emphasis on biological and medical topics. Some medical schools take the standpoint that special premedical physics courses are desirable, but here the view differs as to the sufficiency of a basic general course, or the necessity of a second premedical course in physical instrumentation. And so, although there is unanimity as to the need of a reform in the existing physics courses for premedical students, there is no general agreement as to the means of its achievement.[3]

Considering the great number of colleges and universities involved, agreement can hardly be expected. Premedical training is offered in many institutions, but only a small number provide specifically for premedical students. Their number in the United States and Canada may well lie between 20,000 and 30,000 when we include students of biology, pharmacy, dentistry and agriculture, who are basically interested in the same physics course.[4] They obviously represent only small numbers in the smaller colleges, and hence in most colleges no special course in physics is offered for premedical students.[2,5] However, among physics instructors there is a lack of awareness of the problem, as a result of their preoccupation with other problems, lack of specialized training, and inability to obtain general directives and textbooks.

The American Association of Physics Teachers has taken steps to bring the problem to the attention of its members by setting up a special committee whose initial activity has been to prepare a collection of illustrative material which would allow the use of relevant numerical examples taken from the field of biology.[4] Such material is not otherwise easily available to the physics instructor unfamiliar with biology, physiology and medicine, and the committee's report should provide a wider and more suitable choice of problem subjects than is available exclusively from the fields of engineering and physics. This collection is expected to improve the premedical physics course insofar as it should show the students the importance of physics for the biological sciences, and so stimulate their interest in physics.

If wisely applied by an instructor familiar with some aspects of biology, the collection of illustrative examples may prove of value. At least, the instructor may become so interested in the illustrations that he will make an effort to acquaint himself with the biological background of the subject matter which the numerical examples illustrate. However, it must be kept in mind that at this stage in the premedical student's education, his knowledge of biology is very limited. If the material were used in the same manner as numerical examples are usually utilized in physics courses, its use would have little consequence, and would still justify the following remark:

> The general criticism coming to me over a period of fifteen years has been that all the [physics] instructor does is work problems.[6]

This statement has to be taken with a grain of salt. In fact, quite a number of instructors try to impart to their students more than skill in work-

[1] Herrington, "The expanding role of physics in biology and medicine," *Trans. Royal Soc. Canada* **41** sec. 3, 1 (1947).
[2] Schmitt, "The growing importance of physics in the premedical and medical curriculums," *Archives of Physical Medicine* **28**, 71 (1947).
[3] "Alpha-Epsilon-Delta review of premedical education," *The Scalpel* **17**, August, 1947.
[4] Barnes, "Objectives of the AAPT committee on physics for students of biology and medicine," *Am. J. Physics* **15**, 375 (1947).
[5] Barkow, "Physics in the predental and premedical curriculum," *Am. J. Physics* **16**, 236 (1948).
[6] See ref. 3, p. 138.

ing examples, although it may be safely admitted that the majority emphasize problem work. However, by their problem-type examinations and quizzes they frequently undermine their own better efforts. Hence, it is with much justification that the review from which the previous quotation was taken goes on to say:

> It is a complaint that has come from nearly all medical schools and especially from physiologists who have found their students in medical schools *lacking in understanding of the principles* involved, not only in the nature and use of physiological apparatus, but *in the physical processes* carried on in living systems (italics mine).[7]

This criticism touches the root of the problem; the physics teacher only too often presupposes that principles are understood when a correct answer to a more or less routine numerical problem is given. In fact, the way in which physics examinations are set and the spirit in which they are received by the students, nullify the effect of the instructor's earnest endeavors to give his students some understanding of principles. I outlined recently a method of testing and examining which was intended to correct the shortcomings of the present method, and which is also applicable to the testing of biophysical material.[8]

Cultural Aims

The AAPT Committee on Premedical Physics Courses has limited its activities to the immediate problem of improving the curriculum in one respect, namely, the infusion of biological and medical applications into the introductory physics course for premedical students, apparently without having taken into consideration the broader problem of the introductory physics course as a whole. The serious attention which the AAPT has always given to the improvement of physics instruction at the college level makes it evident that no curtailment of possible cultural content of such courses is intended by recommending the introduction of biophysical subject-matter. However, there is always the danger that a premedical physics course—whether a freshman or a sophomore course—could become another technical subject, adapted to such an extent to the true, or imagined, needs of the medical training and profession, that the achievement of a broader cultural and educational program is in jeopardy. The present physics curriculum, loaded with antiquated material (as it often is) already constitutes an obstacle to the achievement of broader aims. Adding new material in the form of physical biology to the syllabus may easily accentuate the fragmentary character of the traditional physics course, thereby contributing further to the bored indifference of many students.

The requirement of a broad cultural education for the scientist is often mistakenly interpreted as the demand for the introduction of more courses in literature, history and economics into the curriculum, rather than for permeating the science courses with educational and cultural substance.[9] I am much in favor of a thorough education of the scientist in the humanities, but do not close my eyes to the fact that the answer is rather *multum, non multa*, and that humanistic values are implicit in any science course and can well be put in relief. Scientists in general, and physicists in particular, have been reluctant to bring out the cultural contents of their subject. However, the introductory physics course is gradually being conceived by many as an educational course without special emphasis upon the vocational side of training of any of the students. Illustrations from the biological field should therefore be considered as *humanizing elements* in the cultural curriculum and their inclusion in the most elementary physics courses is highly desirable.

On the other hand, whenever a special premedical physics course is offered, it should not neglect to attain a cultural character in the most general sense. Such a course should include the *basic* physical principles, the philosophical and social implications of the physical sciences, the historical development of physics, and, last but not least, the connections between physics and biology. A premedical physics course which is offered as the first and only introduction to physical science cannot be based mainly on the applications of physics to biology and medicine;

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[7] See ref. 3, p. 136.

[8] Blüh, "Physics examinations and the new curriculum," *Am. J. Physics* **16**, 20 (1948).

[9] Blüh, "The contribution of physics to the college curriculum," *Am. J. Physics* **10**, 39 (1942).

it has to take into account the general education of premedical students.

> . . . since [premedical students] . . . are members of society and involved equally with the rest in the development of our general intellectual enterprise.[8,10]

The premedical physics course has, in the first place, to give thorough instruction in theoretical and practical physical methods, indispensable for the study of physiology and parts of medicine. Secondly, it has to show that the physical method can only be applied wisely in biology by those who know the nature of the problem of living matter. Therefore, as a result of stress upon philosophical ideas, students become prepared for problems whose true importance can only be appreciated by increased familarity with the biological sciences. If this task is undertaken in addition to the more general aims mentioned above, it seems doubtful whether the program can be achieved in a single course in premedical physics. The desirability of a second physics course for premedical students is thus indicated, to be offered at a level where they have developed greater maturity of thought and a susceptibility to the wider theoretical problems of science.

A Second Premedical Physics Course

A second course for premedical students has been suggested, with the understanding among physics instructors that it is to be one in *physical instrumentation* rather than in principles. Medical school teachers have been outspoken against such a course, and the editor of a review in *Scalpel* summarizes the discussion on this point as follows:

> . . . it is definite that most medical and premedical educators do not feel that a special course in physics, dealing with instruments, their operation and application, is desirable. The reason most often given is a sound one, namely, that the principles are or should be included in all elementary courses in physics and the actual *uses* in medicine should be left for the medical schools to teach in connection with clinical experience. For the benefit of the premedical teachers of physics who feel that the course is desirable it is urged by many of our respondents that their efforts might be better spent, and would be of more lasting value, if they did a better job of teaching the fundamental laws and principles of physics with enthusiasm, interest, and consideration of the interests of the students.[11]

Although I am fully in agreement with this statement, it should not be allowed to obscure the fact that training in the correct use of physical apparatus and instruments is a very useful discipline. Many physical instruments and types of apparatus are really tools, and the premedical student should certainly acquire experience in their proper handling. The physics laboratory is the place for such training, which would be an essential part of the second physics course for premedicals. For this reason the laboratory for a second course for premedical students has not only to be equipped with the conventional laboratory equipment, but with advanced research apparatus. The lectures can then be devoted to theoretical principles, a coherent outline of the subject, and biological applications.

It may be argued from the standpoint of the colleges that a second premedical physics course should be the responsibility of the medical schools. To establish it in the colleges of arts and sciences has the advantage that many other groups of students of parallel interests can profit from it, such as students of biology, pharmacy, biochemistry, bacteriology, agriculture and other sciences. It has also to be considered that a high percentage of students who intend to enter a medical school or a dental school may not be accepted as students of medicine or dentistry. A number of these will become candidates in the end for degrees in the pure or applied biological sciences, and then may find it possible to obtain a better understanding of physics than is offered in the introductory physics course. Unfortunately, the need for a better and broader knowledge of physics has received less recognition among the disciples of biology than among teachers of medical sciences and clinicians.

The second physics course could be rightly considered as the "premedical physics course" to which the prerequisite may be a physics course usually offered to college freshman. The advanced course in physics for premedical students, students of biology, and others, may also satisfy the interests and needs of senior nonphysics students and of graduates. By meeting this demand a

[10] *Wiley Bulletin* (Wiley and Sons, Inc., New York), May, 1948, p. 5. Discussion remark by Professor L. W. Taylor.

[11] See ref. 3, p. 130.

greater number of students can be enrolled, a factor not without importance in small institutions. However, this makes it imperative to organize the course as an independent unit with its own aims and methods.

Content of a Premedical Physics Course

The general physics course invites the treatment of the whole of physics in the conventional way in which textbooks are written, and in which the instructor tries to get the student acquainted with everything he might need at a later stage. Lack of time prohibits such a procedure in the advanced course for premedical students. Here, the requirement is to teach a limited number of topics with a sufficient degree of exactness, so that they are not left as unrelated scraps and fragments. The student must be made aware at every step in his learning process of the coherence of physical science.

There are obviously various possible ways of taking students through the realm of physics without following a conventional pattern. One way is the historical approach, which will not fail to attract interest and to develop appreciation for the sciences and their achievements. This approach also allows the connections between progress in the biological and the physical sciences to be brought out. In general, it seems that the historical approach—if it does not take on the character of lectures on the history of physics—is better reserved for the introductory physics course. An approach along fundamental lines has been developed by myself and applied to various comprehensive courses in physics and to a second premedical physics course. This approach is based on two fundamental methods of presenting physics: the phenomenological way and the way of forming pictures of processes unobservable by our senses. In practice, it means treating first the physical phenomena of macroscopic nature without investigating closely the possible underlying submacroscopic phenomena, which are then treated as a separate unit.

The phenomenological part of the subject matter can be dealt with independently and unconventionally, as Table I shows. In the first chapter we take up mechanics in such great detail that clarity of the basic concepts is obtained. The second chapter may deal with the various forms of energy, starting from the mechanical equivalent of heat, and treating electrical energy, including the energetic effects of alternating currents, and the energy of radiations. The third chapter is devoted to phenomena in which the concept of field of force is needed, and where gravitational, electric, and magnetic fields are discussed. In the fourth chapter we pass on to the

TABLE I. Biophysical scope of the lecture course.

1. *Methods of science;* mathematical introduction.
2. *Mechanics.* Movements of the limbs, levers, surgical instruments, standing and walking, center of gravity, balance, microbalance. Centrifuge, ultracentrifuge, effects of acceleration, 'g'-suit. Respiration, atmospheric pressure, diver's bell, pneumatic ventilation. Hydrostatic phenomena, balneology, blood circulation (Bernoulli), work of the heart. Ergometer.
3. *Forms of energy.* Calorimetry and animal calorimetry, food values. Heat conductivity, heat radiation. Electric heating, diathermy, electrosurgery. Photometry. Calorimetry. Light therapy, heliotropism, absorption law for light.
4. *Fields of force.* Geotropism. Electric current in medical practice, electromagnet, electromagnetic metal locator, induction coil, string galvanometer, cardiograph.
5. *Fields of waves.* The ear, range of hearing, Weber-Fechner's law, auscultation and percussion, stethoscope, supersonic vibrations. High frequency electric currents, capacitance effects, high frequency therapy. The eye, color vision, stereoscope, optical illusions, refractometer, cystoscope, nephelometer, microscope, Abbé's theory, ultramicroscope, electron microscope. Spectra (infra-red, visible, ultraviolet), bactericidal action, x-rays, tolerance dosage, Bucky-Potter diaphragm, absorption law, mass absorption coefficient, absorption spectra, spectrum of the blood.
6. *Kinetic theory of matter.* Elasticity, bone structure, viscosity, surface tension, osmosis, diffusion, adsorption. Colloids, Brownian movement. Thermometer, katathermometer. Humidity, temperature regulation. Evaporation, anesthetics. Vacuum technique, vacuum stills, dried blood-plasma.
7. *Electrical properties of matter.* Electrolysis, electrophoresis, iontophoresis. Resistance of the human body. Effects of electric currents, electric membrane potentials. Conductivity and dielectric constants of biological fluids, nerve conductivity. Gas ionization. Electronics, cathode-ray oscillograph, cardiograph, encephalograph, electron tubes, electronic voltmeter, electron microscope, betatron, x-rays, effects of radiation.
8. *Atomic and nuclear physics.* Radioactivity, artificial radioactivity, tracers in physiology and medicine, neutron therapy. Biological effect of radiations, mutations.
9. *Thermodynamics.* Efficiency of the human machine. Animal transformer, ergometer. Metabolism.
10. *Physics and biology.* Statistical laws. Mechanism and vitalism. Applications of physics in medicine. Biophysics.

propagation of energy by waves, including a choice of topics from sound, electromagnetic phenomena, and physical optics.

The second part of the course is not conceived as a mere chapter on atomic physics, but as a full account of the kinetic theory and quantum theory, beginning with the atomistic hypothesis, applied to all three states of matter, and to electronic, atomic, and nuclear phenomena. In this division we have ample opportunity for dealing with all the important phenomena in living organisms having physical significance and biological applications. Although it is a temptation to conclude a physics course with nucleonics, the atom bomb, and similar spectacular matters, to do so in a course for biologists would be ill-advised. I think, indeed, that we must prevent the biologist from getting the impression that living matter is nothing but a complex of physical elementary particles. A naïve materialism, not unknown among biologists, does not need to be supported by the physicist.

For this reason we may turn back the thoughts of our premedical students to macroscopic realities by considering selected topics in the field of thermodynamics. Here is an opportunity not only to show the importance of statistical laws and of such biologically relevant topics as metabolism and the "human machine," but also to point out the limitations of the physical approach to biological phenomena. Such controversial topics as that of vitalism-materialism should be included in this part, and dealt with in an objective presentation without attempting a dialectic synthesis of the problem.

The syllabus as outlined above gives opportunity for introducing the main principles of physics and a great number of biological and medical applications which have been entered in Table I. Lacking a suitable textbook, the student must learn to summarize the main points of lectures, and to develop the art of *précis*-writing from the spoken word—very useful accomplishments for the student at this stage.

Mathematical Requirements

Although the arrangement and order of topics is most important for bringing out the coherence of physical thought, it is also important for illustrating the techniques used in the treatment of points of detail. Here a question immediately arises: How much mathematics can safely be administered to a group of students who are conventionally averse to rigid quantitative thinking, and, by the very choice of their vocational training, have decided against overshadowing their studies by complex mathematical calculations? I do not believe that the standard of a physics course is set solely by the mathematical apparatus used, but it is evident that any advanced physics course requires a certain amount of mathematics and that in a second premedical physics course the necessary mathematics may have to be acquired in the course, along with the physical subject matter. Higher standards of mathematical requirements may be very desirable, but any attempt to introduce them will probably meet with opposition.

The same correspondent to the review in *Scalpel* whom we have quoted above, expresses the hope that:

> Some one should be able to develop a course in physics in which methods other than mathematical are used in explaining physical processes.[12]

Although this prayer is on many premedical students' lips, it cannot be easily granted. In physiology, biochemistry, radiology, and other biological sciences so much mathematics is applied today that we should try purposely to give the students some idea of the mathematics they may need later. The question is not simply one of ability to solve problems, since it is sometimes found that many premedical students of high intelligence and considerable capacity fail in any kind of numerical calculation. To segregate premedical students solely on account of mathematical abilities seems to be unjust. The lecture course and the method of examining must therefore be varied and elastic enough to allow those with little mathematical aptitude to display their native abilities.

Numerical examples, I believe, should always grow out of the lecture program and be related to topics under consideration. The laboratory should give ample opportunity for numerical work in connection with practical assignments, so that problem solving for the sake of arithmetical calculations needs to be stressed only moderately.

[12] See ref. 3, p. 138.

Mathematics used in deductions should make use of algebra, graphical representations, and elementary calculus. The more important mathematical functions should become familiar to the students. This can be done by use of a special mathematical introduction, which can be made into a permanent achievement of the student by frequent application.

Techniques of simple differentiation and integration have a good possibility of being grasped when the starting point is taken in analytical geometry. The comprehension of the existence of a relationship between functions and their graphs makes for a better understanding of the functions and graphs themselves. In trying to infuse some calculus into the course, a task, it seems, entirely the responsibility of the physics teacher, we may actually gain very little ground, but achieve a better understanding of various physical statements.

Laboratory Work

An important part of the work of the premedical student should be done in the laboratory, using exercises coordinated with the lectures. The equipment of the laboratory associated with a second premedical course must be such as to give students training in the handling and use of apparatus they will encounter in the physiology laboratory, in clinical and radiological work, and elsewhere. The premedical laboratory is not an exclusive course in instrumentation, and no attempt need be made to obtain familiarity with the techniques of instrument building, constructing elaborate apparatus, and reading technical drawings. On the other hand, the laboratory of the advanced premedical physics course should be free of antiquated and obsolete apparatus of the demonstration type. The cost of equipment is considerable, but the furnishing of the laboratory with real laboratory apparatus and instruments is a general asset to a department, and of importance to other departments and to research programs. It may be pointed out that the order in which experiments are done (see Table II), by following the lecture course closely, allows the progress from simple to more complicated (and more expensive) physical equipment, which includes the analytical balance, hydrostatic balance, centrifuge, surface tension balance, hydrophilic balance, potentiometer, conductivity bridge, photoelectric densitometer, refractometer, microscope, colorimeter, spectroscope, and x-ray machine.

TABLE II. Scope of the laboratory work (in four-week periods).

1. Analytical balance; sensitivity, weighing, densities.
   Hydrostatic balance; specific gravities of liquid mixtures, volume contraction.
   Centrifugal apparatus.
   Pendulum, spiral spring, inertia balance.
2. Mechanical equivalent of heat by an electric method.
   Specific heat by the method of cooling.
   Temperature coefficient of electric conductivity of a metal.
   Absorption of light in glass and colored solutions.
3. Potentiometer, determination of emf of cells, polarization of galvanic cells, thermocouples.
   Impedance and self-induction.
   Characteristic curves (diode and triode).
   X-ray absorption coefficients.
4. Coefficient of heat conductivity, and determination of the exponent in Stefan's law of radiation.
   Refractometers (Abbé, dipping). Specific refractions of solutions.
   Microscopes, magnification, immersion objective, microprojection.
   Colorimetry (Beer's law); adsorption isothermal of dye on charcoal.
5. Sedimentation and diffusion (Brownian movement); use of centrifuge.
   Dielectric constants.
   Absorption spectrum; use of spectrograph.
   Optical polarograph.
6. Viscosity of solutions.
   Surface tension (Du Noüy and film balance).
   Electric conductance of solutions.
   Radioactivity, Geiger counter.

The laboratory work must not consist of routine measurements; instead, a limited piece of research should always be undertaken, yielding a definite result. By emphasizing the research character of some of the assignments interest in the laboratory is greatly increased. Rather elaborate assignments must be studied in preparation for laboratory work of this nature. Concentrated work has to be done during laboratory hours, and the elaboration of notes will take up a considerable part of the time allotted to the total physics work. The laboratory work provides an opportunity for getting thoroughly familiar with various kinds of graphs, especially of logarithmic and semilogarithmic representations. The evaluation of results must include the estimation of errors involved in all readings, and the calculation of the combined error of a result. The description of

apparatus can be reduced to a minimum, but an attempt should be made at good composition. The laboratory should be viewed as a discipline in independent work, order, relevance, and the art of expression.

Taken as a whole, the second premedical physics course, as outlined in the last three sections, can provide for a well-balanced education and preparedness for further studies. The instruments of educational control, tests and examinations, and if possible, personal supervision and individual discussions, must not only be used to keep a check on the student's achievements, but may be used to augment his knowledge and comprehension. Support through able assistants in the laboratory is important. Examples of various types of testing have been given in a recent article.[8]

Speaking from experience over many years with students of varied previous training, age grouping, and interests, it seems not at all doubtful that a number of students attending such a course will be found to be insufficiently prepared, or will lack certain aptitudes, or even show shortcomings in the necessary degree of intelligence, to be able to make full use of the facilities offered to them. Nevertheless, even if good use is made only by a relatively small number, it may be important to give the few, who merit the labor and expenditure involved, a solid basis for scientific studies, and to carry the others up to a fair level of understanding.

Biophysical Training

Some further considerations may be added here, closely connected with the question of premedical training, and concerning the teaching of biophysics, a science which is now developing into a separate discipline. At present biophysics is not yet a well-defined subject in its own right, and is still relegated to be the handmaiden of biology. However, it must be assumed that as a result of an increasing interest in the use of physical methods in biology, biophysicists, or rather physical biologists, will in the future come in greater numbers than before from the ranks of the biologists. This will call for another new educational venture on the part of the physics teacher. Many physics departments will find it desirable to establish divisions for biophysical training and research, and universities may be led to the creation of some counterpart of the interesting organization for biophysical studies set up by the Massachusetts Institute of Technology.[13]

By many students the second premedical physics course may be used as a preliminary to more advanced study in biophysics. Students who have received a training on the biological side will, however, need some further physics and mathematics courses. At the same time, provision will have to be made for advanced physics students to become acquainted with biology. Graduates in physics and biology will also have to be considered in any such new training facilities.

It is often remarked that mathematics and physics should be studied in the first college years, as these subjects, compared with other disciplines, demand a greater power of concentration and a greater mental effort—capacities, which allegedly are more pronounced in the younger than in the older student. On the other hand, it is said that it is easy for a man trained in physics to reach familiarity with biological techniques and ideas. There is probably some truth in the observation, although the number of cases on which any conclusions can be based is small. However, it has to be pointed out that in future more mature biologists will be attracted into the field of biophysics, and we should not make the mistake of expecting them to learn physics by attending the same classes as the younger students of physics. The same may apply in reverse to the study of biology by trained physicists.

Very often the young college student goes through his class work with spells of interest, and passes his examinations in a flash of inspiration. The senior student, whose interests are already focused, needs a more concentrated and relevant form of study course. Lacking the alertness and elasticity of the younger mind, he does not fare well in competition with the younger student. Experience with veterans shows that there is a need for special courses for older men and women in the universities—school teachers constitute one such group—which covers the same subject

[13] "Educational opportunities in chemical biology and physical biology," Department of Biology, Massachussets Institute of Technology, Cambridge 39, Mass.

matter as the normal courses but with a greater measure of integration and appealing to the higher maturity of the students who wish to work in the border-field of biophysics,

> where greater specialization and more integration is the problem and the opportunity of the biophysicist.[14]

The training of the premedical student and of the biophysicist calls for a synthetically-minded integrator and a man with a strong educational interest. The incompetence of the scientific specialist to deal with the wider task of education has sometimes brought discredit upon efforts to broaden the curriculum of the elementary physics courses, and the same may happen if the infusion of biological material into a physics course is incompetently executed. There is good reason to believe that physics instructors are aware of the need for new ways of approach, and that the general realization of integrated studies will lead to an educational achievement of important consequence.

An Educational Experiment

The present paper does not pretend to offer a generally acceptable solution of a controversial question, but is submitted as a contribution in the field. It is partly based on recent experience. A few years ago, at the University of British Columbia, a second physics course for premedical students, and others, was introduced. It is open to all students except physics honor students and students of engineering, who are channeled into specialized courses. The prerequisite for this course is the first year physics course, which is more or less adapted to the engineering students. The majority of the students attending the course register as premedical or predental students and take the course either in their second or their third years. Only one-half of about 160 students finishing the course every year have a chance of being admitted to a medical or dental school. The others take degrees in biology, bacteriology, and in other sciences. Students of pharmacy take the course in their fourth year; students of biology and agriculture have not been conspicuous in their attentance.

For two years I have been trying to arrange the course in the way outlined above. My experience during two main sessions and one summer session bears out the fact that the course can be given with advantage to a group of students showing little uniformity in matters of preparation and age grouping (a high percentage were veterans). The lecture course (3 hours weekly) and laboratory (2 hours weekly) follow approximately the order given in Table I. Tests and examinations of the objective type have been used.[8] The laboratory has been equipped with modern laboratory apparatus with the sympathetic support of Dr. G. M. Shrum, head of the department. The laboratory consists of a special room with space for a maximum of 24 students, and a separate x-ray laboratory and dark room. Three sets of apparatus for each experiment are available and four different experiments can be performed at the same time. Twenty experiments of the 24 given in the table have already been fully organized. Two students work at one experiment at a time; nine laboratory sections and two lecture sections have been necessary; five assistants were employed. Mimeographed laboratory assignments, given out every four-week period, are kept in a special file by the students, and comprise a booklet of about 100 pages.

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[14] Bronk, "The relation of physics to the biological sciences," *J. Appl. Physics* 9, 139 (1938).

RECENT MEETINGS

Chicago Section

A very successful fall meeting was held at the University of Chicago on December 4, 1948, with an attendance of approximately 50 members. Three papers were presented:

Quantitative experiments on gyroscopic motion and rotational inertia. PHILIP A. CONSTANTINIDES, *Wilson Junior College.*

A model of a ferro-magnet. T. H. BERLIN, *Northwestern University.*

The new 170-in. synchrocyclotron. S. K. ALLISON, *Institute for Nuclear Studies, University of Chicago.*

Other features of the meeting included a tour of the University of Chicago physics laboratory and cyclotron building, a lunch in Hutchinson Commons, and a business meeting at which the following officers were elected for 1949: *President*, H. R. Voorhees, *Chicago City Junior College; Vice President*, W. E. Peterson; *Secretary-Treasurer*, P. A. Constantinides, *Wilson Junior College; Representative to AAPT Executive Committee*, L. I. Bockstahler, *Northwestern University.*

LESTER I. BOCKSTAHLER, *Secretary*

Kentucky Section

The annual fall meeting of the Kentucky Section of the American Association of Physics Teachers was held on October 30th at the University of Kentucky. Forty-seven members and guests attended. Professor J. G. Black, Eastern Kentucky State College, presided during the presentation of the following contributed papers:

Some problems in heat conduction in cylindrical coordinates. R. H. RITCHIE and C. B. CRAWLEY, *University of Kentucky.*

Use of dioptric power of lenses in the solution of optics problems. P. C. OVERSTREET, *Morehead State College.*

Construction of a standard lamp. R. HANAU, *University of Kentucky.*

Does a baseball curve? JOSE RUBIO, *Berea College.*

A balanced D.C. amplifier. W. O. SHROPSHIRE, *University of Kentucky.*

Determination of the specific gravity of wood by elementary students. FOSTER BURGESS, *Berea College.*

Construction and study of the characteristics of Geiger-Mueller counters. G. S. HURST, *University of Kentucky.*

Some comments on the relation of science and society. WALDEMAR NOLL, *Berea College.*

Course and curriculum. ERLAND RITCHIE, *Centre College.*

Some comments on returning to teaching from industry. J. G. BLACK, *Eastern Kentucky State College.*

The meeting closed with a luncheon in the Student Union Building.

LEWIS W. COCHRAN, *Secretary*

Southern California Section

The regular fall meeting of the Southern California Section of the American Association of Physics Teachers was held in the Physics-Biology Building at the University of California at Los Angeles on Saturday, October 30, 1948. One hundred and fifteen members and guests attended the morning program, seventy-three attended the luncheon in Kerckhoff Hall, and eighty heard the afternoon program. Between luncheon and the afternoon session most of those at the meeting inspected the cyclotron on the U.C.L.A. campus.

The morning program provided an invited talk by Professor J. R. Richardson of the University of California at Los Angeles on the subject, "Artificial Production of Mesons," followed by the following ten-minute contributed papers:

Entropy—How should it be taught? L. E. DODD, *University of California at Los Angeles.*

An experiment for the study of transients in electrical circuits. V. L. BOLLMAN, *Occidental College.*

Horrid words in physics. FOSTER STRONG, *California Institute of Technology.*

Three-dimensional electrical units QLT and ILT. F. W. WARBURTON, *University of Redlands.*

Physics and our civilization. D. L. SOLTAU, *University of Redlands.*

Education—or merely training? V. Force.—A fact or a fiction? GEORGE FORSTER, *Pasadena City College.*

The fluorescent lamp as an experiment in a.c. vector diagrams. WILLARD GEER, *University of Southern California.*

Lantern slide color demonstration. SHELDON BROWN, *University of California at Los Angeles.*

Report from the high school physics test committee.

The afternoon program consisted of an invited paper: "Microwave Demonstrations in the teaching of physics" by Professor W. H. Pickering, California Institute of Technology.

Following the afternoon program, the members stayed for a business meeting. Professor Soltau reported on the problems being discussed by the Executive Committee of the American Association of Physics Teachers. Mr. Enholm reported for the Teaching Load Committee, saying that the report dealing with teaching loads at the college level would soon be ready, but that great difficulty and complexity were being encountered in evaluating the high school teaching load.

Professor David L. Soltau of the University of Redlands was re-elected as Section representative on the Executive Committee of the American Association of Physics Teachers for the year 1949.

FOSTER STRONG, *Secretary-Treasurer*

ANNOUNCEMENTS AND NEWS

Book Reviews

Introduction to Atomic Physics. S. TOLANSKY. Second Ed., reprinted with an appendix on nuclear fission and atomic energy. Foreword by Sir Lawrence Bragg. Pp. 351+xi. Longmans, Green and Co. London and New York, 1948.

In a comparatively small book, Dr. Tolansky presents a very good account of the development of atomic physics at the level of a junior year course in an American college. There are over 200 separate articles in the text. Whatever faults the book has are essentially minor ones; they are inherent in any attempt to present so many topics in such a small space. The extra-nuclear part of atomic physics occupies about 60 percent of the book; the rest is devoted to nuclear physics.

The book starts with a rather extensive treatment of the discharge of electricity through gases; this is followed by a discussion of the determination of e/m and e of the electron, and by a good chapter on the mass spectrograph. A very good treatment of the older quantum theory is introduced at this stage (Chap. 5); it includes a development of the Rayleigh-Jeans blackbody radiation law and a comparison of this with the observations of Lummer and Pringsheim. Planck's quantum theory is then introduced and the Planck radiation law derived. This is followed by a discussion of the quantum theory of specific heats with a derivation of Debye's equation. There is a short article on the photoelectric effect (a much more complete treatment is given in Chap. 7), and the chapter ends with a discussion of the hydrogen spectrum and the Bohr theory of the hydrogen atom together with a brief mention of wave mechanics.

If the material on wave mechanics (Chap. 11) had been introduced at this point, some of the discussion in succeeding chapters of the thermionic effect and of atomic spectra would have been more meaningful to the reader.

The treatment of optical spectra in Chap. 8 is extremely weak. For example, although the vector model of the atom is used extensively, there is not a single diagram to illustrate how these vectors are to be treated. To quote from the text: "The spin angular momentum, denoted by the quantum number s ($s=\frac{1}{2}$), can combine vectorially with the orbital angular momentum l to form a resultant denoted by the quantum number j, which thus represents the total angular momentum of the electron. The vector addition is usually shown as $j=l+s$, the addition being quantized." Unless details are given concerning the method of quantization, preferably with diagrams, such a statement can have little meaning for the beginner. The Zeeman effect, for example, is given just one short paragraph, and is introduced chiefly to bring in the term, magnetic quantum number, which is needed for the discussion of the Pauli exclusion principle. Similarly, the subject of x-ray spectra is treated much too briefly in Chap. 9. For example, the x-ray energy-level diagram is given for a heavy element, and the values of n, l and j are given for the different energy levels, but there is no discussion of how the values of these quantum numbers are arrived at.

The subject of extra-nuclear physics is completed by an extensive chapter on the structures of crystals, a good chapter on wave mechanics, and a very short chapter on electron collisions in gases in which the Ramsauer effect and the Franck and Hertz experiments on critical potentials are discussed. The rest of the book is devoted to nuclear physics except for a chapter on relativity theory. This section begins with radioactivity and the radioactive transformations, followed by chapters on the alpha-particle, beta-rays, gamma-rays and cosmic radiation. Each of these chapters is fairly complete and the material adequately treated. The chapter on cosmic radiation is one of the best in the book. It contains a good elementary account of the Yukawa theory of the origin of the meson and the properties of the meson as known before 1945.

Chapter 18, The Nucleus of the Atom, is much too brief to do justice to all of the topics mentioned therein. It contains a discussion of Gamow's theory of alpha-particle disintegration (no mention is made of the work of Condon and Gurney), of Bohr's theory of nuclear disintegration, of artificial disintegration of nuclei and artificial radioactivity, of the nuclear origin of stellar energy (no mention of Bethe in this connection), and of nuclear spins. Some amends have been made by the addition of an appendix on nuclear fission and atomic energy. The inclusion of more numerical data on such things as the half-life periods in induced radioactivity, and of nuclear reaction energies for those cases treated, would greatly improve this chapter.

There are a few misstatements in the text. For example, on p. 66, . . . "some elements, like arsenic and iodine, have only a single isotope, whilst others have many, mercury possessing nine and tin eleven." Mercury has only seven stable isotopes and tin has ten stable isotopes. In discussing characteristic x-ray spectra, p. 154, the author states "The K-radiation of every element consists of two lines (each of which is actually complex)." A glance at Fig. 9.6 shows that there are three lines in the K series of the heavier elements. For the very light elements the statement quoted is meaningless.

As mentioned earlier, the faults of this book are minor ones. Dr. Tolansky's book is a very welcome addition to the textbooks on atomic physics at the intermediate level. It is recommended to physicists and chemists who desire a brief and authoritative treatment of the very rapidly expanding field of atomic physics.

HENRY SEMAT
The City College of New York

Outlines of Physical Chemistry. FARRINGTON DANIELS. Pp. 713+viii, Figs. 164, 15×23 cm. John Wiley and Sons, Inc., New York, 1948. Price $5.00.

Those who have read *Outlines of Theoretical Chemistry*, published under the authorship of Frederick H. Getman in 1913, and have followed in order the revisions leading up to the seventh edition published in 1944 under the

names Getman and Daniels, may not have been conscious of any revolutionary changes in the text. Each edition is a modernized version of the preceding edition. When *Outlines of Physical Chemistry* by Farrington Daniels, published in 1948, is compared with the seventh edition of the Getman and Daniels text again no startling change appears. However, if the latest text is compared with those published before Professor Daniels began his revisions in 1930, one is impressed by the number of changes in subject matter, arrangements and methods of presentation which have been made. It is true that the chapter headings are similar, but the figures, indexes, references to literature and methods of expression are so different that the new book is immediately recognized as the work of Professor Daniels and no longer a revision of a Getman text.

Since many more people are familiar with the seventh edition published in 1944 than with earlier editions, all subsequent comparisons will be made with that edition.

From the point of view of binding, paper, printing and other details in mechanical production, the publishers have produced a good book. One is immediately struck by the greater ease with which the new text is read. There are fewer letters per line and fewer lines per page. All of the impression of crowded pages felt when using the seventh edition is lacking.

Each figure is accompanied by an explanatory paragraph which makes its use easier and more effective than that of the previously merely numbered figures whose explanations were in the text.

Some subject matter has been moved from one chapter to another. Appropriate material on atomic and molecular forces is collected in a new chapter, the third in the book. Other related matter, much more difficult, such as calculation of activation energies, including Morse curves, is placed in the appendix. The third law of thermodynamics is treated immediately after the first and second laws, instead of in a chapter entitled Chemical Thermodynamics which included a group of topics such as free energy calculations, activities, partial molal quantities, and entropy by graphical methods. This chapter was not included in the new text. Its sections were rewritten and put into other chapters with which they were logically related, or in a few cases relegated to the appendix.

My desk copy of the seventh edition carried several marginal notes at points at which it seemed necessary to warn a student that the statements of the text could be misunderstood. A high fraction of such notes would be unnecessary with the new textbook, for the author has clarified the statements beyond possible misunderstanding. A few statements could yet be improved. On p. 36, HCl is cited as an example of the covalent type of linkage though on p. 494 the reaction of HCl with H_2O to form H_3O^+ and ^-Cl is recorded. On p. 212, the reference to Figs. 51 and 52 is to figures in the seventh edition. On pp. 576, 577 and 578 the description is incomplete for the apparatus shown in the accompanying Fig. 133. If elements 57 to 71 inclusive are the rare earth group, that group includes 15 elements, as it must to fill the numbers between barium and hafnium; on p. 623 this textbook goes along with almost all others and says there should be 14 rare earth elements, then states that 13 of these are known and implies that hafnium is the fourteenth rare earth element needed to complete the group.

However, it requires much reading and close scrutiny to find any errors, misprints, or inadequately expressed ideas in this volume. The arrangement is more logical than that of its predecessor. The tables of constants have been brought down to date. References up to 1947 are cited. The number of problems is slightly larger for the 21 chapters than it was for the previous 23 chapters. Every set of problems has been reorganized, even when the topics treated in the chapter were not changed. Reorganization includes: rearrangement, new data in old problems, new problems on old principles and problems on new topics.

The new *Outlines of Physical Chemistry* by Farrington Daniels lacks few, if any, of the features which made 'Getman and Daniels' popular. The subject matter is actually arranged in more logical order. The book is timely; its treatment of topics is modern; it includes much new material.

F. E. Brown
Iowa State College

New Members of the Association

The following persons have been made *members* or *junior members* (*J*) of the American Association of Physics Teachers since the publication of the preceding list [Am. J. Physics **17**, 50 (1949)].

Adams, Clifford L., Missouri School of Mines and Metallurgy, Rolla, Mo.
Anderson, William R., 4519 Altgeld St., Chicago 39, Ill.
Bishop, Dwight E. (*J*), 1301 Ruby Ave., Houghton, Mich.
Brennen, Robert W. (*J*), Box 903, University of Houston, Houston, Tex.
Brophy, James J., Jr. (*J*), 1904 Nordica Ave., Chicago 35, Ill.
Bry, John C. (*J*), 746 Beechwood Drive, Hovertown, Pa.
Buelteman, Herbert Oliver, Jr. (*J*), 1509 E. Houghton, Houghton, Mich.
Bushner, Matt G., 2072 Campus Rd., Toledo 6, Ohio.
Cahill, John J., Jr. (*J*), 362 Carrol St., Akron 4, Ohio.
Collins, Royal Eugene (*J*), Apt. 3, Bldg. 33, University of Houston, Houston, Tex.
Davis, Dr. Kenneth E., Reed College, Portland 2, Ore.
Durst, Rev. Gerald F., Pontifical College Josephinum, Worthington, Ohio.
Gates, Halbert F., Berea College, Berea, Ky.
Goldberg, Leon P. (*J*), 4219 18th Ave., Brooklyn 18, N. Y.
Goldhammer, Paul (*J*), 3014 N.E., 30th St., Portland, Ore.
Hafner, Emelyn, 2004 N. Kickapoo, Shawnee, Okla.
Hare, Robert R., Jr. (*J*), 3F Observatory Court, Box 415, Greencastle, Ind.
Harrison, Paul W., Jr., 6108 Kimbark Ave., Chicago 37, Ill.
Hendrickson, Thomas J., Physics Department, M.C.M. & T., Sault Ste. Marie, Mich.
Hipp, Fred C., Howard University, Washington, D. C.
Kington, Leason K. (*J*), 148 University Ave., Buffalo, N. Y.

Kruschwitz, Walter M., Cumberland University, Lebanon, Tenn.
Lehmann, Ernest Henry (*J*), 36 Dellwood Rd., Eggertsville 21, N. Y.
Li, My C. W. (*J*), California Institute of Technology, Pasadena 4, Calif.
Lindgren, Ralph G. (*J*), 1301 Ruby Ave., Houghton, Mich.
Little, Mrs. G. K., 325 Queen City Ave., Tuskaloosa, Ala.
Loeffler, Sister M. Constance, Mount Marcy College, 3333 Fifth Ave., Pittsburgh 13, Pa.
Luck, Clarence F. (*J*), 147 University Ave., Buffalo, N. Y.
Margerum, Robert F. (*J*), 6656 Ave. "O," Houston 11, Tex.
Marlin, Arnold (*J*), University of Buffalo, Buffalo, N. Y.
McClurg, Glenn O., Illinois Institute of Technology, Chicago 16, Ill.
Millman, George H., Box 558, State College, Pa.
Muldawer, Dr. Leonard, Temple University, Philadelphia 22, Pa.
Nielsen, Dr. Alvin Herborg, Department of Physics, University of Tennessee, Knoxville, Tenn.
Ortiz, Eddie, Physics Department, College Station, Tex.
Prisk, Bert Charles (*J*), 200 Hubbell St., Houghton, Mich.
Randall, Charles A., Randall Laboratory of Physics, University of Michigan, Ann Arbor, Mich.
Riedel, R. Atherton, Oklahoma A. & M., Stillwater, Okla.
Roth, Nathanial, 2461 Davidson Ave., Bronx 63, N. Y.
Sandberg, Herbert J., 133 W. 69th St., New York 23, N. Y.
Schillbach, Dr. Horst M., 1602 Houghton Ave., Houghton, Mich.
Shandley, Paul D. (*J*), 200 Hubbell St., Houghton, Mich.
Shuler, Marion P., Jr. (*J*), 109 First St., N.E., Washington, D. C.
Stanley, Gordon Lee (*J*), 106 Upland Rd., Houghton, Mich.
Van Nostrand, Robert, 8 Cronin Court, Rolla, Mo.
Wegner, Harvey E., 837 Rainier Hall, University of Washington, Seattle, Wash.

AMERICAN JOURNAL OF PHYSICS VOLUME 17, NUMBER 3 MARCH, 1949

LETTERS TO THE EDITOR

English Composition and American Prose

I HOPE you will see fit to print this letter, if only to encourage your readers to see both sides of a problem and to adhere to tradition as long as that tradition is worthy. I was much interested in the article on "The Responsibility of the Teacher of College Physics for the Student's Facility in American Prose" by P. I. Young in your issue of November, 1948, and agreed with most of it. Miss Young adheres to the phrase "English Composition" but distinguishes between English prose and American prose. I write as an Englishman, though long domiciled in Canada. What she says applies much to the Canadian student, for although the official language in English-speaking Canada is English, defined by our highest authorities as that prescribed by the Concise Oxford Dictionary, yet by and with frequent intercourse between Canada and the U.S.A. the phrases known as Americanisms have crept into the common life of Canada. The student finds it difficult to break away from the many good-humoured colloquialisms of daily life when he should use more dignified prose in his university reports and in the physics papers he may write for journals.

In English there is a standard—the King's English—but who sets the standard in America? Once we start to speak and to write loosely, the downward path is taken and quick is the descent. Facility in speaking and in writing good English is an art of the highest kind, and few possess the gift (I wish I had it). But for all that, we need not mangle our heritage of good literature. Under the section *Diction* on p. 426, Miss Young uses "practice" as a verb. This makes an Englishman shrink with horrors To him "practice" is the noun and "practise" is the verb. Americans use these two words indiscriminately, and I have seen even in the same article both used correctly and incorrectly. Again on p. 426, Miss Young says "It behooves the teacher." Now why the extra *o* in behooves? I know "hooves" as the plural of "hoof," but I say "behoves" when I mean "behoves." To shorten words as much as possible, the American has removed the *u* from honour, colour, vapour and such like, and then perversely adds letters to make behooves and skillful.

The Englishman has become familiar with the change of spelling of centre to center, centimetre to centimeter, etc., in American prose and he just breathes a sigh of pity for people who use such terms; but when he meets sulfur, and fosforus, he is properly horrified. Also he is astonished at the change of the pronunciation of data, water, apparatus, and research to datta, watter, apparattus and *re*search. In one laboratory textbook by a great American, I read the phrase "Get the demonstrator to O.K. the connections." Now where is this to stop? The American may say "Never" —but if that is so then truly will American prose differ from English prose.

The American physicist, as evidenced by articles in the *American Journal of Physics*, seems obsessed with the renaming of physics units and terms. We use the cgs system in the laboratory. It works well, for we handle grams and measure by centimetres. But now we are asked to change to the mks system, though we rarely have to measure a length greater than 1 m or handle a mass greater than 1 km (I use the abbreviations to avoid the spelling which you wouldn't like). Once we were used to gauss as the unit of magnetic field strength and to Maxwell as the unit of magnetic flux. Now America asks us to replace these terms by the Oersted and the Weber, respectively. Confusion upon confusion! Meteorologists the world over use the "bar" for 10^6 dynes per cm^2, for a pressure of 1000 millibars is about equal to the average atmospheric pressure and we can conveniently express the changing atmospheric pressures as so many millibars without going beyond the decimal point. Along comes the G.E.C. of Schenectady

and redefines the "bar" as 1 dyne per cm². Where is this infernal tinkering of words and units to end?

University of Toronto, Toronto 5, Canada

JOHN SATTERLY

The "Practical" System of Electromagnetic Units

I HAVE read with interest the recent articles in the *American Journal of Physics* dealing with systems of electromagnetic units,[1] and would like to add my bit to the discussion. The choice of the number and nature of our primary quantities depends, of course, principally on the purpose to be accomplished, and need not be restricted to any one viewpoint. However, I believe that all that is essential in our systems of electromagnetic units can be encompassed in two groups: (1) A flexible "practical" system in terms of four basic units; (2) A simpler theoretical system in terms of two basic units.

A "practical" system need not be regarded as synonymous with the mks system. The usual version of the mks system selects as basic the same two textbook experiments which have traditionally served as the basis for the cgs electrostatic and electromagnetic systems, but in place of a field constant of one, we have quantities impractical both in magnitude and dimensions. The simplicity of the Gaussian system is lost with no compensating gain. Actually, definitions expressed directly in terms of the force between two charges, or between two currents, are relics of the outmoded *action at a distance* viewpoint, and are of no great help in interpreting the field itself. If we are to introduce dimensions into our field structure, our aim should be to provide a simple, practical tool for the interpretation of this structure. Quantities appropriate to this purpose would appear to be the field velocity c, and the field impedance k or Z_0 (376.7 ohms). No other field constants are needed.

In setting up the dimensional structure of our practical units, the theorist could well take a hint from the practical man. Volts and ohms have long served as primary units in laboratories and practical work, even though it may not have been fully realized that these units are equally suitable for the formulation of our basic equations. If we regard the volt, ohm, and second as primary units, our field equations may be put into a symmetrical form applicable to any arbitrary unit of length, certainly an enormous practical advantage. Further, we may adapt our units to other orders of magnitude by selecting a suitable decimal subdivision of our time coordinate. Thus we might use a millionth or a billionth of a second for the short wave region, or perhaps a billion-billionth of a second for atomic magnitudes. The coulomb, farad, weber, and henry are reduced in the same proportion. The ordinary practical electrical units form a suitable basis for all these systems, exemplifying the great flexibility and practicality of a four-dimensional (four-unit) basis without the disadvantages and limitations of the customary mks system. We can make our practical units really practical.

Transition to units of the cgs systems is readily accomplished. We set the field impedance k equal to 4π or to unity, to give a system of either the Gaussian or Lorentz type. By setting $c=1$, electrostatic and electromagnetic derivations are merged into one system. The system becomes essentially two-dimensional, and is completely determined in terms of two basic units, which we may take to be the erg and the centimeter.

The formulation of the field equations in terms of these systems of units is discussed in some detail in my article "The Electromagnetic Field" in the *Journal of the Western Society of Engineers*.[2]

4849 W. Belden Ave., Chicago 39, Illinois

GUSTAVE R. HOLM

[1] F. W. Warburton, *Am. J. Physics* **16**, 435 (1948); J. A. Eldridge, *Am. J. Physics* **15**, 390 (1947).
[2] G. R. Holm, *J. West. Soc. Engrs.* **53**, 87 (1948).

Subharmonic Resonance

THE recent description by C. A. Ludeke[1] of an interesting mechanical device for demonstrating subharmonic resonance in a nonlinear system raises the question of whether the same phenomenon might easily appear in an analogous electrical circuit. The accompanying data indicate that such is the case, and that the experiment described in my paper on a nonlinear circuit[2] could be extended readily to show subharmonic generation.

The circuit (Fig. 1 of my earlier paper) consists of the series combination of an iron-core inductor, a fixed capacitor, and an oscillator. An oscilloscope is used to observe the waveform of the voltage across the inductor. If the oscillator is set to supply a signal of suitable frequency and amplitude to the circuit, it is found that the voltage across the inductor contains two component frequencies. One component is at the frequency of the oscillator; the other is at one-third this frequency, and is of somewhat smaller amplitude. The pattern on the oscilloscope is easily recognizable as the sum of two frequencies, however. The characteristics of the driving voltage needed to produce the subharmonics are shown in Fig. 1. Only such combinations of frequency and amplitude as are represented by points in the shaded area allow the formation of the subharmonic. Even with a suitable driving signal, production of the subharmonic requires proper initial conditions. These conditions may be obtained by random opening and closing of the circuit with the voltage applied. Starting conditions seem less critical at higher frequencies and voltages.

Also plotted in the figure is a curve of the voltage across the inductor at the subharmonic frequency, as measured with a wave analyzer. Points on this curve correspond to those on the curve of driving voltage, with the unusual property that the smaller driving voltage gives the larger subharmonic voltage. The voltage across the inductor the oscillator frequency is very nearly the same as the driv-

ing voltage, and is not shown. Equation (12) of Ludeke's paper may be modified so as to allow calculation of the subharmonic voltage. The modified equation is $E_s=2^{-\frac{1}{2}}(\omega/3)\phi_s=2^{-\frac{1}{2}}(\omega/3)[(\omega^2/9-\omega^2{}_0)/\frac{3}{4}b]^{\frac{1}{2}}$, where E_s is the rms voltage of the subharmonic. Constants for the present circuit, taken from my paper, are $\omega_0{}^2=1.39\times10^5$ and $\frac{3}{4}b=9.4\times10^7$. A curve calculated from this equation is shown in Fig. 1, and is seen to agree only very roughly with the measured data.

The operation of the circuit is evidently sufficiently complicated that the assumptions used in deriving the

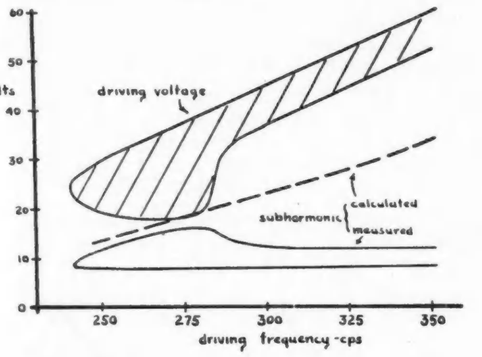

FIG. 1. Driving voltage necessary to produce subharmonic resonance.

above equation are not valid. For example, the presence of a subharmonic at frequencies as low as 180 cycle sec^{-1} is predicted by the equation, while the observed lower limit is about 240 cycle sec^{-1}. A large current exists in the actual circuit at the oscillator frequency, while this current is ignored in the theory. Because of this difference between theory and actual conditions, a modification of Ludeke's Eq. (13) is of no use in finding the necessary driving voltage. The quantitative results of measurements on the electrical circuit are disappointing, but the circuit does present a simple means of demonstrating the formation of subharmonic oscillations.

Yale University
New Haven, Connecticut

W. J. CUNNINGHAM

[1] C. A. Ludeke, *Am. J. Physics* **16**, 430 (1948).
[2] W. J. Cunningham, *Am. J. Physics* **16**, 382 (1948).

Subharmonic Resonance; a Reply

AN interesting electrical analogue for demonstrating subharmonic resonance is presented by Mr. Cunningham, but he is incorrect in expecting his experimental results to agree with Eq. (12) of my recent paper[1] on this subject. The difference in the results obtained by his analogue and by mine is due to the fact that in my experiments the subharmonic frequency predominates while in

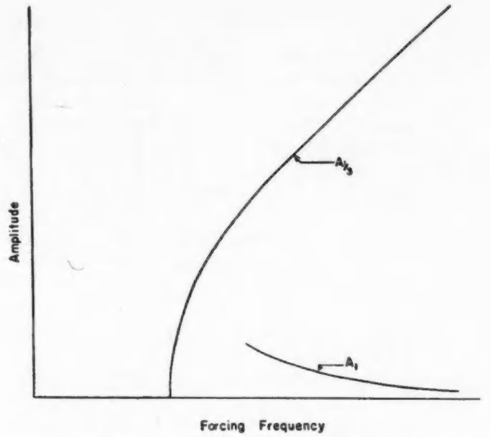

FIG. 1. Amplitudes A_1 and $A_{1/3}$ as functions of the forcing frequency.

his experiments the forced frequency predominates. Consider for example Eq. (5) of my paper,[1]

$$m(d^2x/dt^2)+k_1x+k_3x^3=Po\sin\omega t, \tag{1}$$

and assume it has a solution

$$x=A_{1/3}\sin\omega t/3+A_1\sin\omega t. \tag{2}$$

Upon substituting Eq. (2) into Eq. (1) we find

$$\begin{aligned}(-A_{1/3}\omega^2m/9&+k_1A_{1/3}+3k_3A_{1/3}{}^3/4+3k_3A_{1/3}A_1{}^2/2)\sin\omega t/3\\&+(-A_1\omega^2m+k_1A_1-k_3A_{1/3}{}^3/4+3k_3A_{1/3}{}^2A_1/2\\&+3k_3A_1{}^3/4)\sin\omega t-(3k_3A_{1/3}{}^2A_1/2)\cos(2\omega t/3)\sin\omega t\\&-(3k_3A_{1/3}A_1{}^2/2)\sin(\omega t/3)\cos2\omega t-(k_3A_1{}^3/4)\sin3\omega t\\&=Po\sin\omega t.\end{aligned} \tag{3}$$

If now we consider $3k_3A_{1/3}{}^2A_1/2$ negligible compared to the other coefficients and also $A_{1/3}$ to be larger than A_1, then $3k_3A_{1/3}A_1{}^2/2$ and $k_3A_1{}^3/4$ are also negligible and Eq. (3) can be written in the form:

$$\begin{aligned}(-A_{1/3}\omega^2m/9&+k_1A_{1/3}+3k_3A_{1/3}{}^3/4)\sin\omega t/3\\&+(-A_1\omega^2m+k_1A_1)\sin\omega t=Po\sin\omega t.\end{aligned} \tag{4}$$

If Eq. (4) is to be true for all values of t we have:

$$k_1-\omega^2m/9+3k_3A_{1/3}{}^2/4=0, \tag{5}$$

and

$$A_1=\frac{Po}{k_1-m\omega^2}. \tag{6}$$

Equations (5) and (6) are plotted in Fig. 1, and obviously can be of value only if $A_{1/3}$ is larger than A_1. If now we extend this case to its limit by allowing A_1 to approach zero, Eqs. (5) and (6) are replaced by Eqs. (12) and (13) of the original paper.[1]

If, however, $A_1>A_{1/3}$, as is the case in Mr. Cunningham's experiments, the above arguments are invalid and a theoretical procedure similar to that discussed in reference 4 of the original paper[1] must then be utilized.

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[1] C. A. Ludeke, *Am. J. Physics* **16**, 430 (1948).

A Physics Tour

IN 1942, some members of the Physics Department of the Univeristy of New Hampshire conceived the idea of going about the state to spread the "gospel of science." The Extension Service sent to the principals of high schools a letter stating our "wares." The response from 56 schools was greater than anticipated and three stops per day were scheduled in order to reach all before the end of the term. Two faculty members, using a university truck, went out for a week. The following week two other members relieved them, after which the first pair resumed the lectures. In all, four faculty members participated.

We used a school assembly period of about 45 minutes (net). During that time we demonstrated the gyroscope functioning as a compass and as a stabilizer for the monorail car and the boat tending to rock. The precession experienced by a whirling propeller subject to a torque was shown. The possibility of making an egg stand on end *without breaking it* (Columbus had to break it) was commented upon. One boy produced a hard-boiled egg for us to prove our statements. Volunteer student assistants helped with the rocking boat and propeller experiments, making up for their inexperience by giving to the audiences a greater feeling of participation in the experiments.

The magnetic "girdle" was demonstrated by passing a toy ship over a photoflash bulb "bomb," which did not discharge when the girdle carried a suitable current. The talk ended with a plea for interest in and understanding of mathematics. Movies were not used, since this service was regularly available through the Extension Service.

We were received with interest and, in some cases, enthusiasm. Although our schedule did not permit time to visit with the students as we would have preferred, on several occasions, as the years have passed, students in my classes have said that they witnessed the demonstrations and became interested in physics to a greater extent as a result of the talks. We have felt well rewarded for our efforts.

In planning a workshop or a traveling exhibit, it would be well to remember that all students might not be interested in the topic of the day, atomic energy alone; other phases of physics should be included. Quite a bit of time is required to travel as well as to set up and take down the equipment, so that two schools is the maximum which can be visited in one day with any degree of satisfaction.

Experience of other college teachers of physics in attempting to meet and interest high school students by this method have undoubtedly brought forth similarly gratifying response, which should be of interest to readers of this column.

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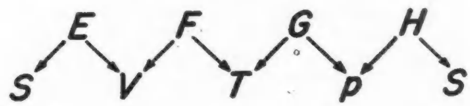
Maxwell's Thermodynamic Relations

CONTRARY to the opinion of Mr. C. M. Focken,[1] as given in your November issue I am convinced that nobody should memorize Maxwell's thermodynamic relations. The expressions to memorize are the differentials of the internal energy E, the free energy $F=E-TS$, the Gibbs free energy $G=E-TS+pV$ and the enthalopy or heat function $H=E+pV$:

$$dE=TdS-pdV$$
$$dF=-SdT-pdV$$
$$dG=-SdT+Vdp$$
$$dH=TdS+Vdp.$$

These expressions are the core of thermodynamics. A mnemonic aid is the following diagram expressing the relation between dependent and independent variables:



Maxwell's relations follow immediately from the fact that the differential expressions are total differentials. From the expression for dE, for instance, it is immediately concluded that:

$$\frac{\partial^2 E}{\partial S\partial V}=\left(\frac{\partial T}{\partial V}\right)_S=-\left(\frac{\partial p}{\partial S}\right)_V=\left(\frac{\partial^2 E}{\partial V\partial S}\right).$$

Anybody who knows the expressions for dE, dF, dG and dH by heart, or better still, their extension after introduction of the chemical potentials μ:

$$dE=TdS-pdV+\mu dn, \text{ etc.}$$

should be able to reproduce the whole mathematical skeleton of thermodynamics without counsulting a textbook.

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[1] *Am. J. Physics* **16**, 450 (1948).